Lecture 25

Thursday, February 23, 2023 12:31 PM

* Questions

$$\int_{-1}^{2} \frac{x}{1+x^2}\,dx = \int_{2}^{5} \frac{x}{u}\,\frac{du}{2x} = \int_{2}^{5} \frac{du}{2u} = \frac{1}{2}\ln u\Big|_2^5$$

$$= \frac{1}{2}\ln\frac{5}{2}.$$

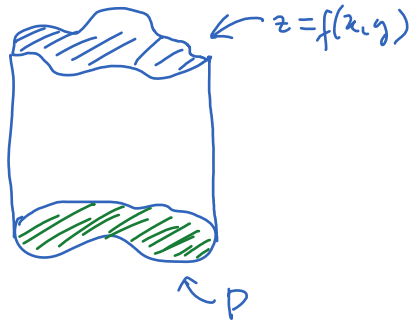
$u = 1+x^2$

$du = u'dx = 2x\,dx$

$dx = \dfrac{du}{2x}$

x	-1	2
u	2	5

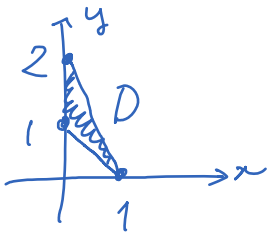
* Volume of a solid above a general region.



If $D = \{(x,y): a \le x \le b,\ h(x) \le y \le g(x)\}$ then

$$\iint_D f(x,y)\,dA = \text{vol}(E) = \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx$$

Ex Find the volume of the solid under the plane $z = x+y$ and above the triangle D.



$$D = \{(x,y) : 0 \le x \le 1,\ -x+1 \le y \le -2x+2\}$$

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{-x+1}^{-2x+2} (x+y)\,dy\,dx$$

$$= \int_0^1 \left(xy + \frac{y^2}{2}\right)\Big|_{y=-x+1}^{y=-2x+2} dx$$

$$= \int_0^1 \left[x(-2x+2) + \frac{(-2x+2)^2}{2} - \left(x(-x+1) + \frac{(-x+1)^2}{2} \right) \right] dx$$

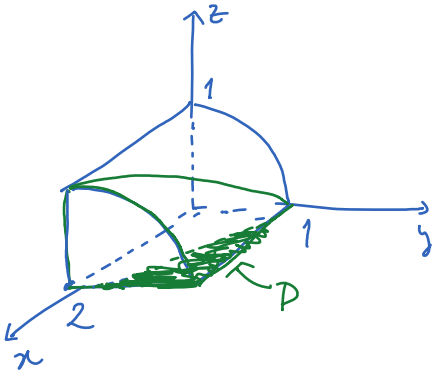
$$= \cdots$$

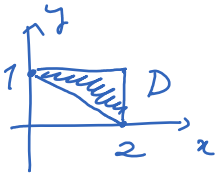
$$= \frac{2}{3}$$

On Mathematica, use the command

Integrate[x+y, {x, 0, 1}, {y, -x+1, -2x+2}]

Ex Find the volume of the solid above the triangle with vertices at $(2,0,0)$, $(0,0,0)$, $(0,1,0)$ and under the cylinder $z = \sqrt{1-y^2}$.





$$D = \{(x,y) : 0 \le y \le 1,\ 2-2y \le x \le 2\}$$

$$\text{volume} = \iint_D \sqrt{1-y^2}\,dA = \int_0^1 \int_{2-2y}^{2} \sqrt{1-y^2}\,dx\,dy = \int_0^1 2y\sqrt{1-y^2}\,dy$$

Let $u = 1 - y^2$

$du = u' dy = -2y\,dy$

$dy = \frac{du}{-2y}$

y	0	1
u	1	0

$$\int_0^1 2y\sqrt{1-y^2}\,dy = \int_1^0 2y\sqrt{u}\,\frac{du}{-2y}$$

$$= \int_0^1 \sqrt{u}\,du = \frac{u^{3/2}}{3/2}\Big|_0^1 = \frac{2}{3}$$