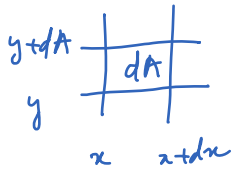
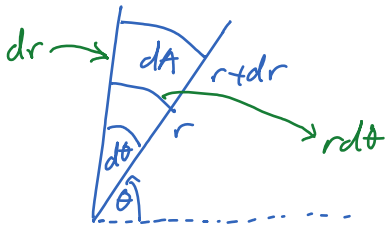
Lecture 27

Monday, February 27, 2023 8:21 AM

* Questions....

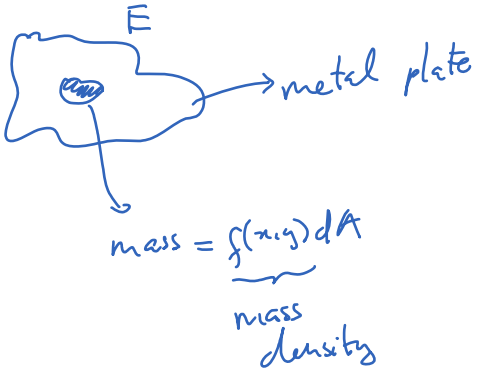
Why is $dA = r\,dr\,d\theta$?





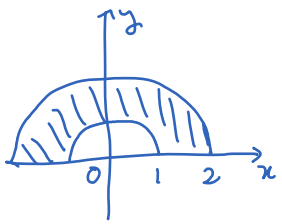
$dA \approx r\,dr\,d\theta$

* Application of double integral:



Total mass is $\iint_E f(x,y)\,dA$

Ex



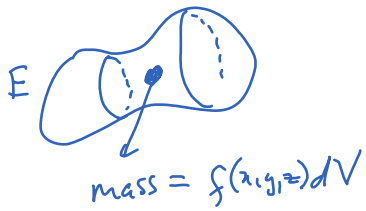
mass density: $f(x,y) = y$

$$\text{Total mass} = \iint_E y\,dA = \int_0^{\pi}\int_1^2 r\sin\theta\; r\,dr\,d\theta$$

$$= \int_0^{\pi} \sin\theta\,d\theta \int_1^2 r^2\,dr = -\cos\theta\Big|_0^{\pi} \frac{r^3}{3}\Big|_1^2$$

$$= 2\left(\frac{8}{3} - \frac{1}{3}\right) = \frac{14}{3}.$$

Mass of a solid.

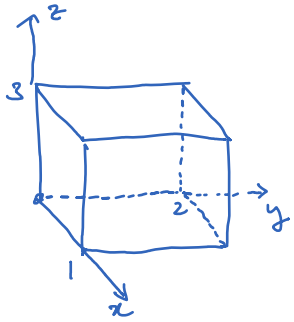


Total mass = $\underbrace{\iiint_E f(x,y,z)\,dV}_{\text{triple integral}}$

If E is a rectangular box $[a_1,b_1]\times[a_2,b_2]\times[a_3,b_3]$ then

$$\iiint_E f(x,y,z)\,dV = \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} f(x,y,z)\,dz\,dy\,dx$$

Ex



mass density : $f(x,y,z) = x+y+z$

$$\text{Total mass} = \int_0^1\int_0^2\int_0^3 (x+y+z)\,dz\,dy\,dx$$

$$= \int_0^1\int_0^2 \left(xz+yz+\frac{z^2}{2}\right)\Big|_{z=0}^{z=3} dy\,dx$$

$$= \int_0^1\int_0^2 \left(3x+3y+\frac{9}{2}\right) dy\,dx$$

$$= \int_0^1 \left(3xy+\frac{3y^2}{2}+\frac{9}{2}y\right)\Big|_0^2 dx$$

$$= \int_0^1 (6x+6+9)\,dx$$

$$= \int_0^1 (6x+15)\,dx = (3x^2+15x)\Big|_0^1 = 18$$