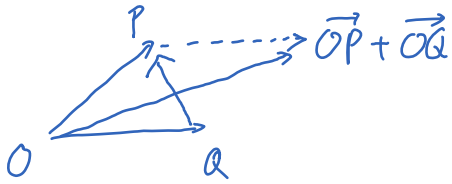
Lecture 3

Wednesday, January 11, 2023 10:14 PM

Vectors



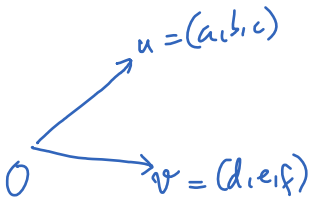
$$\vec{OP}-\vec{OQ}=\vec{QP}$$

$|\vec{OP}|$ = norm / length / magnitude / modulus of vector $\vec{OP}$

= distance from O to $P(a,b,c)$

$$=\sqrt{a^2+b^2+c^2}$$

Dot product



$u\cdot v = ad+be+cf$: dot product of two vectors is a number

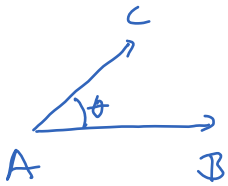
Question

Is $u\cdot v = v\cdot u$?

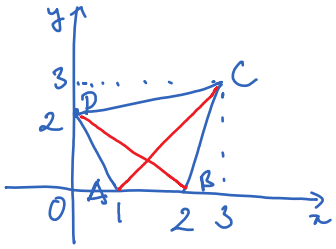
Is $u\cdot(v+w) = u\cdot v + u\cdot w$?

Is $(u+v)\cdot w = u\cdot w + v\cdot w$?

Is $(u\cdot v)\cdot w = u\cdot(v\cdot w)$?



$$\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|}$$ (Law of Cosine in triangle ABC)

Ex 

what is the angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$?

Application

- u and v are parallel if $\cos\theta = 0$. In other words, $u\cdot v = |u||v|$
- u and v are perpendicular if $\cos\theta = 0$. In other words, $u\cdot v = 0$

Ex Find x such that the vectors

$$u = (1,2,2), \quad v = (1,0,x)$$

makes an angle $45°$, $90°$, $180°$, $0°$ with each other.

$$|u| = \sqrt{1^2+2^2+2^2} = 3$$

$$|v| = \sqrt{1^2+0^2+x^2} = \sqrt{1+x^2}$$

$$u\cdot v = 1(1)+2(0)+2(x) = 1+2x$$

$$\cos\theta = \frac{u\cdot v}{|u||v|} = \frac{1+2x}{3\sqrt{1+x^2}}$$

- $\theta = 0°$ $\quad \cos\theta = 1$

$$1+2x = 3\sqrt{1+x^2} \rightsquigarrow (1+2x)^2 = 9(1+x^2) \rightsquigarrow 5x^2-4x+8=0$$

no solution

• $\theta = 90^\circ$ $\quad \cos\theta = 0$

$1+2x=0 \rightsquigarrow x = -\frac{1}{2}$

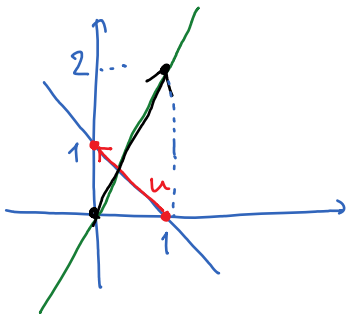
• $\theta = 45^\circ$ $\quad \cos\theta = \frac{\sqrt{2}}{2}$

$$\frac{1+2x}{3\sqrt{1+x^2}} = \frac{\sqrt{2}}{2} \rightsquigarrow \frac{(1+2x)^2}{9(1+x^2)} = \frac{1}{2} \rightsquigarrow 9(1+x^2) = 2(1+2x)^2$$

$$\rightsquigarrow x^2 - 8x + 7 = 0$$

$$\rightsquigarrow x = 1 \text{ or } x = 7$$

Ex Find the angle made between two lines $x+y=1$ and $2x-y=0$.



Vector u connects $(1,0)$ to $(0,1)$.

$u = (0,1) - (1,0) = (-1,1)$.

Vector v connects $(0,0)$ to $(1,2)$.

$v = (1,2) - (0,0) = (1,2)$

$$\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{-1+2}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}}$$

$\theta \approx \ldots$