

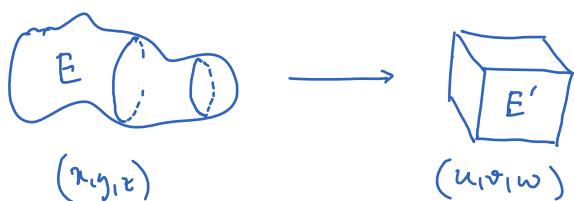
Lecture 32

Tuesday, March 7, 2023 12:29 PM

* Questions

* Change of variables for triple integral

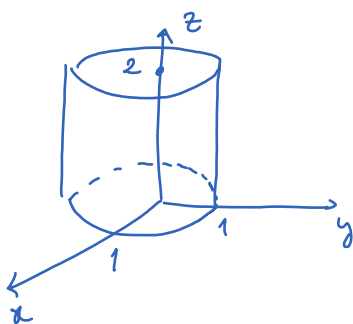
$$\iiint_E f(x, y, z) dV = \iiint_{E'} f(x(u, v, w), y(u, v, w), z(u, v, w)) J dV'$$



$J = \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right|$ is the Jacobian of the change of variables.

* Two common changes of variables

- Cylindrical coordinates suits with cylindrical shapes.



$$\iiint_E x^2 dV$$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$

$$(x, y, z) \rightarrow (r, \theta, z)$$

$$\left. \begin{array}{l} 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \\ 0 \leq z \leq 2 \end{array} \right\} E' = [0, 1] \times [0, 2\pi] \times [0, 2]$$

$$J = r \text{ (we skip the computation)}$$

$$\iiint_E x^2 dV = \iiint_{E'} r^2 \cos^2 \theta r dV' = \int_0^2 \int_0^{2\pi} \int_0^1 r^3 \cos^2 \theta dz dr d\theta$$