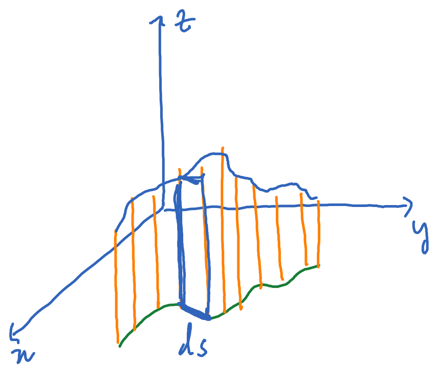
Lecture 36

Tuesday, March 14, 2023 3:53 PM

* Questions...

The line integral $\int_C f(x,y)\,ds$ can be interpreted as the area of the wall built on top of the curve.



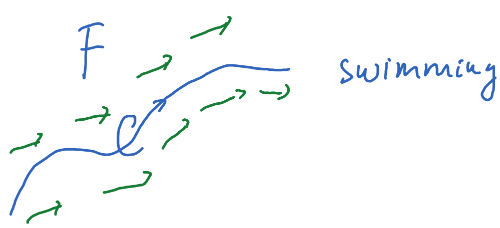
We divide the curve into small pieces.

On top of each piece is a slat of height $f(x,y)$. Area of each slat is

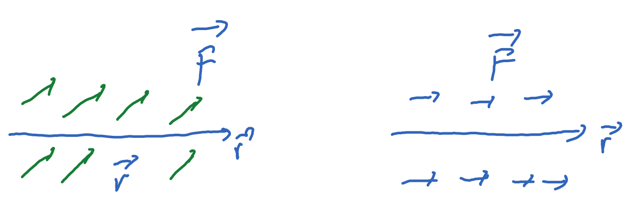
$$f(x,y)\,ds$$

Total area is $\int_C f(x,y)\,ds$

* Line integral of a vector field:



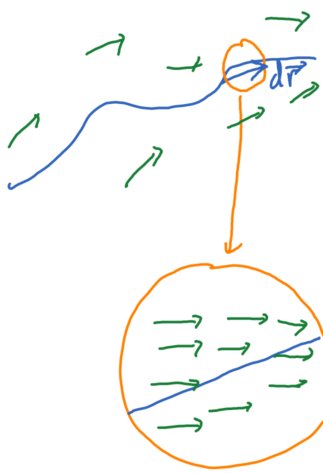
The current can help or oppose you. To quantify this concept, we introduce the work done by a force.



The work depends on the magnitude of $\vec{F}$, of the displacement vector $\vec{r}$, and the angle between them.

work $= |\vec{F}||\vec{r}|\cos\theta = \vec{F}\cdot\vec{r}$

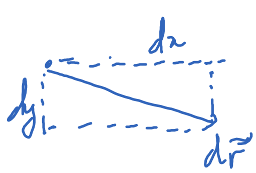
If the vector field is not constant and the trajectory is not straight then we use the zoom-in technique



work $= \vec{F}\cdot d\vec{r}$

Total work $= \int_C \vec{F}\cdot d\vec{r}$

What is $d\vec{r}$?



$d\vec{r} = (dx, dy)$

$\vec{F} = (P, Q)$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C (P, Q)\cdot(dx, dy) = \int_C P\,dx + Q\,dy$$

$$= \int_a^b P x'\,dt + Q y'\,dt = \int_a^b (P x' + Q y')\,dt$$