

Lecture 37

Thursday, March 16, 2023 3:53 PM

* Question ---

Two types of line integrals

$$\int_C f(x,y) ds$$

- Used to find the total mass of a curve or the area built on top of the curve
- Doesn't depend on the orientation of the curve

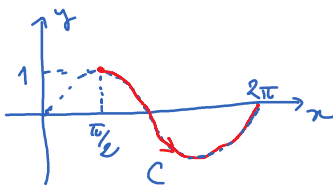
$$\int_C \vec{F} \cdot d\vec{r}$$

- Used to find work, circulation of a force field along the curve
- Depends on the orientation of the curve.



$$\int_{C^-} \vec{F} \cdot d\vec{r} = - \int_C \vec{F} \cdot d\vec{r}$$

E_n



Find the work done by the force field $\vec{F} = (y, x)$ along the curve $y = \sin x$ from $x = \frac{\pi}{2}$ to $x = 2\pi$.

Parametrization of the curve

$$\begin{cases} x = t \\ y = \sin t \end{cases} \quad \frac{\pi}{2} \leq t \leq 2\pi$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C (y, x) \cdot (dx, dy) = \int_C y dx + x dy = \int_{\pi/2}^{2\pi} (y x' + x y') dt \\ &= \int_{\pi/2}^{2\pi} (\sin t + t \cos t) dt \end{aligned}$$