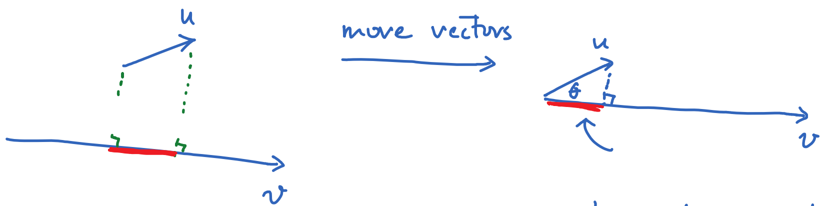
Lecture 4

Friday, January 13, 2023 8:14 AM

* Questions

Dot product $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$

Besides angles, dot product can be used to find the project of one vector onto another vector. This is used a lot in physics: projecting the Newton's second law equation on two different directions.



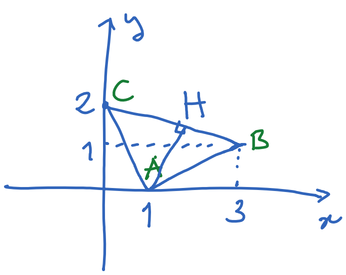
the red bar $= |u| \cos\theta = |u| \dfrac{u \cdot v}{|u||v|} = \dfrac{u \cdot v}{|v|}$

$= u \cdot \left(\dfrac{v}{|v|}\right) = u \cdot e_v$

unit vector in the v-direction

projection of u onto vector v is $u \cdot e_v$.

Ex



Find AH (the altitude of triangle ABC).

CH = projection of vector $\overrightarrow{CA}$ onto vector $\overrightarrow{CB}$.

$$\left.\begin{array}{l} A(1,0) \\ B(3,1) \\ C(0,2) \end{array}\right\} \quad \begin{array}{l} \overrightarrow{CA} = (1,-2) \\ \overrightarrow{CB} = (3,-1) \end{array}$$

$$CH = \overrightarrow{CA} \cdot e_{\overrightarrow{CB}} = (1,-2) \cdot \frac{(3,-1)}{\sqrt{10}} = \frac{5}{\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{2}}.$$

By Pythagorean theorem :

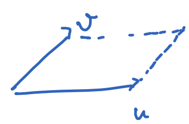
$$AH^2 = AC^2 - CH^2 = 1^2 + (-2)^2 - \frac{5}{2} = 5 - \frac{5}{2} = \frac{5}{2}$$

Thus, $AH = \sqrt{\frac{5}{2}}$.

* Cross product.

Cross product of two 3D vectors is a 3D vector (not a number as with the dot product). For this reason, cross product carries more information than dot product.

Geometric interpretation



$u \times v$ is a vector perpendicular to both u and v, oriented according to the right hand rule, and with length equal to the parallelogram formed by u and v.

Algebraic interpretation

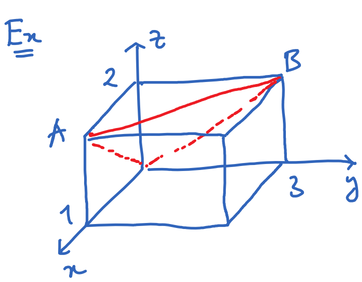
$u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$

$$\begin{array}{ccccc} u_1 & u_2 & u_3 & u_1 & u_2 \\ v_1 & v_2 & v_3 & v_1 & v_2 \end{array}$$

$$u \times v = (u_2 v_3 - v_2 u_3,\ u_3 v_1 - v_3 u_1,\ u_1 v_2 - v_1 u_2)$$

Ex



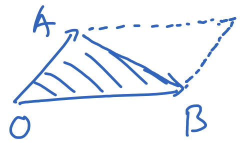
area of red triangle = ?

$\overrightarrow{OA} = (1, 0, 2)$ $\quad 1 \quad 0$

$\overrightarrow{OB} = (0, 3, 2)$ $\quad 0 \quad 3$

$\overrightarrow{OA} \times \overrightarrow{OB} = (-6, -2, 3)$

area of parallelogram $= |\vec{OA} \times \vec{OB}| = \sqrt{6^2+2^2+3^2} = 7$



area of triangle $= \frac{1}{2}$ area of parallelogram $= \frac{7}{2}$.