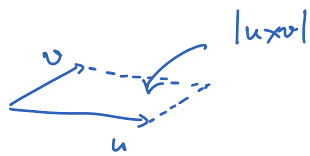
Lecture 5

Friday, January 13, 2023 8:14 AM

* Question ..

Cross product:



Special case. if u and v are parallel then $u \times v = 0$.

vector $(0,0,0)$

u and v are parallel if they have the same or opposite direction.

$$u \parallel v \iff u = tv \quad (\text{for some } t \in \mathbb{R})$$

$$\iff u \times v = 0$$

$$\iff \frac{u_1}{v_1} = \frac{u_2}{v_2} = \frac{u_3}{v_3}$$

$$u \perp v \iff u \cdot v = 0.$$

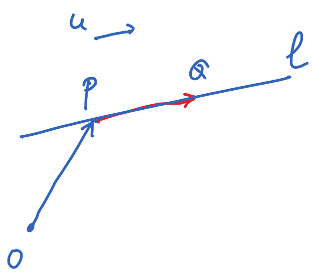
<u>Equation of lines</u>

there are many ways to write the equation of a line.

To identify a line, we need one of the following information:

- a point and a direction
- two points

Suppose a line l passes through $P(a,b,c)$ with direction $u = (u_1, u_2, u_3)$.



A point $Q(x,y,z)$ lies on the line l if and only if $\overrightarrow{PQ} \parallel u$.

That is, $\underbrace{\overrightarrow{PQ}}_{(x-a,\, y-b,\, z-c)} = \underbrace{tu}_{= (tu_1,\, tu_2,\, tu_3)}$

Thus,
$$\begin{cases} x = a + tu_1 \\ y = b + tu_2 \\ z = c + tu_3 \end{cases} \quad (t \in \mathbb{R})$$

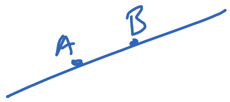
This is a $\underbrace{\text{parametric equation}}$ of the line.

a "constructive" equation that tells us how to takes points on the equation

<u>Ex</u> Eq. of line passing through $P(1,2,3)$ with direction $u = (1,-1,2)$.

$$\begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}$$

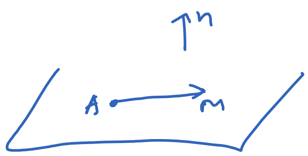
Ex Eq of a line passing through two points $A(1,2,3)$, $B(0,-1,2)$.



$$u = \overrightarrow{AB} = (-1,-3,-1)$$

$$\begin{cases} x = 1-t \\ y = 2-3t \\ z = 3-t \end{cases}$$

Equation of a plane



$A(a,b,c)$: point on plane

$n = (n_1, n_2, n_3)$: normal vector

$M(x,y,z)$: point on plane

$$M \in \text{plane} \iff \overrightarrow{AM} \cdot n = 0 \iff (x-a,\, y-b,\, z-c) \cdot (n_1, n_2, n_3) = 0$$

$$\iff \underbrace{n_1(x-a) + n_2(y-b) + n_3(z-c) = 0}_{\text{general equation of the plane}}$$