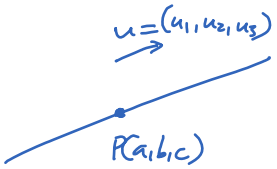
Lecture 6

Thursday, January 19, 2023 8:30 AM

* Questions

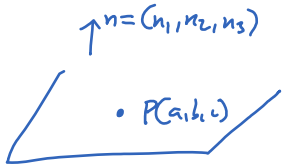
* Parametric eq. of a line

$$\begin{cases} x = a + u_1 t \\ y = b + u_2 t \\ z = c + u_3 t \end{cases}$$



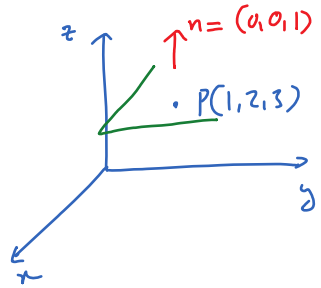
This equation also applies for 2D: $\begin{cases} x = a + u_1 t \\ y = b + u_2 t \end{cases}$

* General equation of a plane:



$$n_1(x-a) + n_2(y-b) + n_3(z-c) = 0$$

Ex. find equation of the plane passing through $P(1,2,3)$ and orthogonal to the z-axis.



$n = (0,0,1)$ is a normal vector

Equation of the plane:

$$0(x-1) + 0(y-2) + 1(z-3) = 0$$

or $\boxed{z = 3}$.

Ex find equation of the plane that contains the point $P(1,2,3)$ and the z-axis.

Normal vector: $\vec{k} \times \vec{OP} = (0,0,1) \times (1,2,3) = (-2,1,0)$.

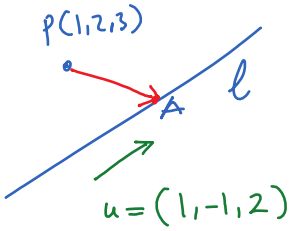
0 0 1 0 0
1 2 3 1 2

Equation. $-2(x-1) + 1(y-2) + 0(z-3) = 0$.

or $-2x + y = 0$

Ex Equation of the plane passing through $P(1,2,3)$ and contains the line

$$\ell: \begin{cases} x = -2 + t \\ y = 1 - t \\ z = 2 + 2t \end{cases}$$



$t=0$ $A(-2,1,2)$ belongs to ℓ

$\vec{PA} = (-3,-1,-1)$

Normal vector $\cdot$ $\vec{PA} \times u = (-3,5,4)$.

$$\begin{matrix} -3 & -1 & -1 & -3 & -1 \\ 1 & -1 & 2 & 1 & -1 \end{matrix}$$

Equation of the plane: $-3(x-1)+5(y-2)+4(z-3)=0$

or $-3+5y+4z=19$.

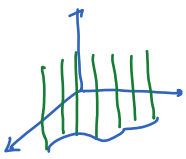
Ex: equation of the line passing through $P(1,2,5)$ and perpendicular to the plane

$$x+y+z=1$$

Surfaces

Cylinder: equation missing one variable

- $f(x,y)=0$
- $f(y,z)=0$
- $f(z,x)=0$



circular cylinder
elliptic cylinder
parabolic cylinder

Quadratic surfaces:

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fzx+Gx+Hy+Iz+J=0$$

This boils down to several basic shapes:

$Ax^2+By^2+Cz^2=1$, $A,B,C>0$

ellipsoid

$z^2=Ax^2+By^2$

two-sided cone

$z=\sqrt{Ax^2+By^2}$: one-sided cone

$z=Ax^2+By^2$, $A,B>0$ elliptic paraboloid

$z=Ax^2-By^2$, $A,B>0$ hyperbolic paraboloid

$Ax^2+By^2-Cz^2=1$, $A,B,C>0$ elliptic hyperboloid (one sheet)

$Ax^2+By^2-Cz^2=-1$, $A,B,C>0$ elliptic hyperboloid (two sheets)

Practice drawing on Mathematica using ContourPlot3D.