Lecture 7

Friday, January 20, 2023 10:25 AM

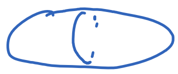
* Questions. .

* Quadratic surfaces:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0$$

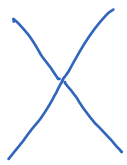
This boils down to several basic shapes:

$Ax^2 + By^2 + Cz^2 = 1$, $A, B, C > 0$



ellipsoid

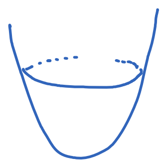
$z^2 = Ax^2 + By^2$



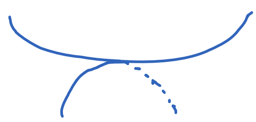
two-sided cone

$z = \sqrt{Ax^2 + By^2}$: one-sided cone

$z = Ax^2 + By^2$, $A, B > 0$ elliptic paraboloid



$z = Ax^2 - By^2$, $A, B > 0$ hyperbolic paraboloid



$Ax^2 + By^2 - Cz^2 = 1$, $A, B, C > 0$ elliptic hyperboloid (one sheet)

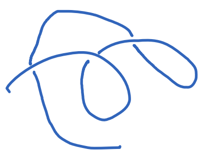
$Ax^2 + By^2 - Cz^2 = -1$, $A, B, C > 0$ elliptic hyperboloid (two sheets)

Practice drawing on Mathematica using ContourPlot3D.

<u>Space curves</u>

Any function $r : [a, b] \to \mathbb{R}^3$ represents a curve in space.

$\searrow$ this is a vector-valued single variable function



Plot on Mathematica using the command ParametricPlot3D.

<u>Ex</u> $r(t) = (\cos t, \sin t, t)$

ParametricPlot3D[{cost, sint, t}, {t, 0, 15}]

<u>Limit</u> $r(t) = (f_1(t), f_2(t), f_3(t))$

$$\lim_{t \to t_0} r(t) = \left(\lim_{t \to t_0} f_1(t), \lim_{t \to t_0} f_2(t), \lim_{t \to t_0} f_3(t) \right)$$

<u>Derivative</u>

$$r(t) = (f_1(t), f_2(t), f_3(t))$$

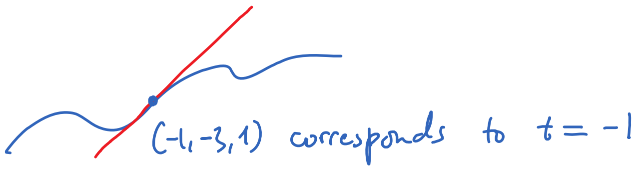
$$r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h} = \lim_{h \to 0} \left(\frac{f_1(t+h) - f_1(t)}{h}, \frac{f_2(t+h) - f_2(t)}{h}, \frac{f_3(t+h) - f_3(t)}{h} \right)$$

$$= (f_1'(t), f_2'(t), f_3'(t))$$

Geometrically, $r'(t)$ is a direction vector to the tangent line of the curve at t.

Ex $\quad r(t) = (t, 2t-1, t^2)$

Find an equation for the tangent line at $(-1, -3, 1)$.



$r'(t) = (1, 2, 2t)$

$r'(-1) = (1, 2, -2)$: this is a direction vector of the tangent line

$$\ell : \begin{cases} x = -1 + t \\ y = -3 + 2t \\ z = 1 - 2t \end{cases} \qquad (t \in \mathbb{R})$$