Lecture 8

Monday, January 23, 2023 10:58 AM

* Questions ..

* Curves $r(t) = (f_1(t), f_2(t))$: curve in 2D

$r(t) = (f_1(t), f_2(t), f_3(t))$ curve in 3D

On Mathematica. use the command ParametricPlot for 2D curves, and ParametricPlot3D for 3D curves.

A curve, from a mathematical point of view, is a single-variable vector-valued function $r\ [a,b] \to \mathbb{R}^2$ or $\mathbb{R}^3$.

<u>Limit</u> $\lim_{t\to a} r(t) = \left(\lim_{t\to a} f_1(t),\ \lim_{t\to a} f_2(t),\ \lim_{t\to a} f_3(t)\right)$

Taking the limit of a vector function is the same as taking the limit of each component of that function.

Ex $\lim_{t\to 0}\left(\frac{t}{\sin t}, \frac{e^t-1}{t}, t+1\right)$

$\lim_{t\to 0}\left(\frac{t}{\sin t}, \frac{\cos t-1}{t^2}, \frac{1}{t^2}\right)$

<u>Derivative</u>

$r(t) = (f_1(t), f_2(t), f_3(t))$

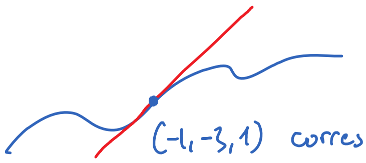
$r'(t) = \lim_{h\to 0} \frac{r(t+h)-r(t)}{h} = \lim_{h\to 0}\left(\frac{f_1(t+h)-f_1(t)}{h}, \frac{f_2(t+h)-f_2(t)}{h}, \frac{f_3(t+h)-f_3(t)}{h}\right)$

$= (f_1'(t), f_2'(t), f_3'(t))$

Geometrically, $r'(t)$ is a direction vector to the tangent line of the curve at t.

Ex $r(t) = (t, 2t-1, t^2)$

Find an equation for the tangent line at $(-1, -3, 1)$.



$(-1,-3,1)$ corresponds to $t=-1$

$r'(t) = (1, 2, 2t)$

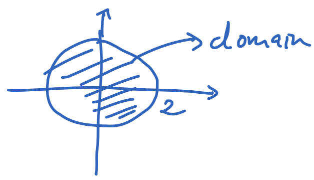
$r'(-1) = (1, 2, -2)$: this is a direction vector of the tangent line

$$\ell.\ \begin{cases} x = -1 + t \\ y = -3 + 2t \\ z = 1 - 2t \end{cases} \qquad (t \in \mathbb{R})$$

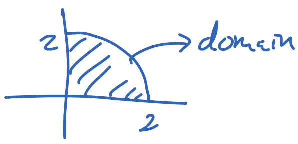
Functions of more than one variable

$$\begin{cases} \text{domain} \\ \text{range} \\ \text{graph} \\ \text{level set} \end{cases}$$

Ex $f(x,y) = \sqrt{4 - x^2 - y^2}$



$f(x,y) = \sqrt{x} + \sqrt{y} + \sqrt{4 - x^2 - y^2}$



Plot using the command Plot3D.

c-level set is the set $\{(x,y) : f(x,y) = c\}$, also called a level curve.

If f has 3 variables, say $f = f(x,y,z)$ then a level set of f is also called a level surface.

Plot level sets using the command ContourPlot.

Ex $f(x,y) = \sin(xy)$

$f(x,y) = \sin(x-y)$

$f(x,y) = (1-x^2)(1-y^2)$

$f(x,y) = \dfrac{x-y}{1+x^2+y^2}$

Limits of a multivariable function

$\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$ if $f(x,y) \to L$ as $x \to a$ and $y \to b$ in any path.