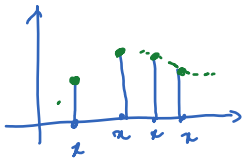
Lecture 9

Tuesday, January 24, 2023 8:10 AM

* Questions -

Calc I: $y = f(x)$, graph is a curve on a plane



$r : [a,b] \to \mathbb{R}^2$ or $\mathbb{R}^3$ is a function of one variable

$f(x,y)$ is a function of two variables

Ex. $f(x,y) = x+y$, xy, x^y, $\frac{x}{y}$, $2x + 5e^y$, ...

Graph of f is a surface in 3D space.

Basic topics about functions

- domain
- range
- graph
- level set

Ex $f(x,y) = \sqrt{9 - x^2 - y^2}$

Domain is $\{(x,y) : f(x,y) \text{ is well-defined}\}$

$= \{(x,y) : 9 - x^2 - y^2 \geq 0\} = \{(x,y) : x^2 + y^2 \leq 9\}$

= the disk of radius 3, centered at (0,0)

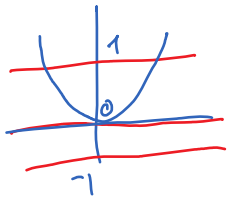
This function can be graphed using Mathematica

```
Plot3D[ Sqrt[9-x^2-y^2], {x,-3,3}, {y,-3,3}]
```

Range of f is $[0,3]$.

Level sets:

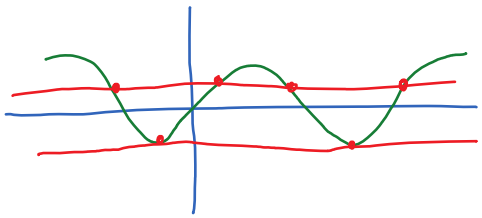
For $f(x)=x^2$, a c-level set is $\{x \quad f(x)=c\}$



1-level set $= \{x : x^2=1\} = \{\pm 1\}$

0-level set $= \{x : x^2=0\} = \{0\}$

-1-level set $= \{x : x^2=-1\} = \emptyset$



$f(x) = \sin x$

Every level set is either an infinite set or an empty set.

Plot level sets on Mathematica use the command ContourPlot

```
ContourPlot[ x^2+y^2==1, {x,-1,1}, {y,-1,1}]
```

```
ContourPlot[ x^2+y^2, {x,-1,1}, {y,-1,1}]
```

(contour map)