Midterm: Some problems for review

The exam will be taken at the Testing Center (Zabel Hall 112) from Feb 13 to Feb 15. The material covered is Section 10.1 - 11.5. This is a closed book exam, 60 minutes long. A 4” x 6” handwritten double-sided note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.

You should review the homework problems, quizzes, examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Perform basic operations on vectors: addition, scaling, dot product and cross product.
- Find the equations of lines and planes.
- Find the domain of a multivariable function.
- Find limits and determine the continuity of a multivariable function.
- Compute partial derivatives.
- Find tangent planes to surfaces and linear approximation of a function.
- Differentiate a function using the chain rule.

Additional problems to practice:
In Problems 1-7, u, v, w are vectors in 3D. Determine whether the statement is true or false. Give reason for your answers.

1) $|-2u| = 2|u|$

2) $|u \times v| \leq |u||v|$

3) $|u \cdot v| \leq |u||v|$

4) $u \cdot v = v \cdot u$

5) $u \times v = v \times u$

6) $(u \times v) \times w = u \times (v \times w)$

7) $(u \times v) \cdot u = 0$

8) The vector $(3, -1, 2)$ is parallel to the plane $6x - 2y + 4z = 1$.

In Problems 11-15, $r(t)$ is a vector function of single variable. Determine whether the statement is true or false. Give reason for your answers.

9) The curve $r(t) = (0, t^2, 4t)$ is a parabola.

10) The curve $r(t) = (2t, 3 - t, 0)$ is a curve passing through the origin.

11) The projection of the curve $r(t) = (\cos 2t, t, \sin 2t)$ onto the xz-plane is a circle.

In Problems 16-20, classify the given surfaces (cylinder/ ellipsoid/ elliptic paraboloid/ hyperbolic paraboloid/ etc).

12) In $\mathbb{R}^3$, the graph of $y = x^2$ is a/an ____________.

13) The set of points $\{(x, y, z)|x^2 + y^2 = 1\}$ is a/an ______________.

14) In $\mathbb{R}^3$, $x^2 + 4y^2 + z^2 = 1$ is the equation of a/an ______________.

15) The set of points $\{(x, y, z)|x^2 + 4y^2 - z = 0\}$ is a/an ______________.

16) The set of points $\{(x, y, z)|x^2 - 4y^2 - z = 0\}$ is a/an ______________.

Write solutions to the following problems.

17) Write the equation of the plane passing through $(2, 1, 0)$ and parallel to $x + 4y - 3z = 1$.

18) Write the equation of the plane passing through $(3, -1, 1)$, $(4, 0, 2)$, $(6, 3, 1)$.

19) Find the area of the triangle with vertices at $(3, -1, 1)$, $(4, 0, 2)$, $(6, 3, 1)$.

20) Write the equation of the plane passing through $(1, 2, -2)$ and containing the line $x = 2t, y = 3 - t, z = 1 + 3t$.

21) Find the equation of the plane perpendicular to the curve $r(t) = (t \cos t,\ t \sin t,\ t)$ at the point $(0, \pi/2, \pi/2)$.

22) Write the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at point $(1, -2, 1)$.

23) A function $f(x, y)$ satisfying $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = f(x_0, y_0)$ is said to be ____________ at (x_0, y_0).

24) Find the region where the function $f(x, y) = \frac{e^x+e^y}{e^{xy}-1}$ is continuous.

25) By Clairaut's Theorem, a smooth function $f(x, y)$ has at most ________ different partial derivatives of third order.

26) Find linear approximation of $f(x, y) = x^3 - 2xy^2$ around $(1, 1)$.

27) If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b), then $\lim_{(x,y)\to(a,b)} f(x, y) = L$. True or false?

28) $\lim_{(x,y)\to(1,1)} \frac{2xy^2}{x^2+y^2} =$ ______

29) $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2} =$ ______

30) $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} =$ ______

Solution keys:

1) True
2) True
3) True
4) True
5) False
6) False
7) True
8) False
9) True
10) False
11) True
12) parabolic cylinder
13) circular cylinder
14) ellipsoid
15) elliptic paraboloid
16) hyperbolic paraboloid
17) $x + 4y - 3z = 6$
18) $-4x + 3y + z + 14 = 0$
19) $\frac{\sqrt{26}}{2}$
20) $6x + 9y - z = 26$
21) $-\frac{\pi}{2}x + \frac{\pi}{2}y + z = \frac{\pi}{2} + \frac{\pi^2}{4}$
22) $z = 8x + 4y + 1$
23) continuous
24) Everywhere in $\mathbb{R}^2$ except for the x-axis and the y-axis
25) 4
26) $f(x, y) \approx -1 + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) = x - 4y + 2$
27) False. Can you give an example?
28) 1
29) 0
30) DNE