Quiz 3 $_{2/27/2023}$

1. Determine all the critical points of the function $f(x, y) = x^2y - 2x - y$.

2. Use the Second Derivative Test to classify the critical points found above into local minimum, local maximum, saddle point, or inconclusive.

3. Let $f(x,y) = x^2 - xy + y^2$ and let D be the unit disk centered at the origin. You are given the following information:

The origin is the only critical point of f.

$$\min_{\text{b.d.}D} f = \frac{1}{2}, \text{ attained at} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$
$$\max_{\text{b.d.}D} f = \frac{3}{2}, \text{ attained at} \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

What are $\min_{D} f$ and $\max_{D} f$, and where are they attained?