Lab 1

In this lab, you will learn how to:

- do basic arithmetic on numbers,
- define functions and graph them,
- find derivatives of a function.

The next lab will be built upon this lab, so please make sure that you go through all the instruction carefully and do all the assignments.

1 To turn in

Do Problems 1-31 in a single Mathematica Notebook file, or ipynb file if you use JupyterLab. Write *your name* and *lab number* at the beginning of your report. Clearly label each problem to separate them from other problems. Make sure to comment on each problem. If your code doesn't run correctly, explain what you are trying to do. Failed code without any comment/explanation will receive 0 point. Submit on Canvas both the *pdf file* and the *source file* (nb or ipynb).

Problems	Points
$\fbox{1-3, 6, 7, 9-13, 16-18, 20-22, 25, 28, 29}$	1
[4, 5, 8, 14, 15, 19, 23, 24, 26, 27, 30, 31]	2
Readability of your report	3
Total: 31	Total: 46

2 Basic arithmetic

- (1) Type (1+2)²⁻³*a followed by Shift+Enter.
- (2) Type a=5 followed by Enter. Then type (1+2)²⁻³*a followed by Shift+Enter.
- (3) Type 35/6, then Shift+Enter. Do it give you a fraction or a decimal-point number?
- (4) Type (1+2)² followed by Shift+Enter. Then type [1+2]² followed by Shift+Enter. What can you learn from this experiment?
- (5) Type N[35/6] followed by Shift+Enter. Then type N[35/6,10] followed by Shift+Enter. Then type N[35/6,20] followed by Shift+Enter. What does the command N[...] do?
- (6) The number $\sqrt{2}$ in Mathematica is Sqrt[2] (notice the capitalized S). Type Sqrt[2] followed by Shift+Enter. Does it give you a decimal-point number?
- (7) Type N[Sqrt[2]] followed by Shift+Enter. An equivalent way of writing this is Sqrt[2] // N
- (8) Find $\sqrt{2}$ using 10 significant digits.
- (9) Type Sin[Pi] followed by Shift+Enter. Next, try the same command but with lowercase S and/or P. Is Mathematica case sensitive?
- (10) Type 47¹⁰⁰; (with the semicolon) followed by Shift+Enter. Then type 47¹⁰⁰ (without semicolon) followed by Shift+Enter.

The semicolon is to hold the output. One uses it when output is too long or not of interest. You may have noticed that the function N is to evaluate a numerical value of an expression. Each function's name is capitalized is followed by a pair of square brackets $[\ldots]$ instead of parentheses (\ldots) as we normally write on paper. For example, the function $\sin(x)$, $\cos(x)$, $\tan(x)$, $\exp(x)$, $\ln(x)$ are written as Sin[x], Cos[x], Tan[x], Exp[x], Log[x] respectively in Mathematica.

- (11) The number e is typed as Exp[1] or E. Find the value of e using 10 significant digits.
- (12) Find the value of $\ln 2$ using 10 significant digits.
- (13) Use a suitable property of logarithm to find $\log_2 3$ using 10 significant digits.
- (14) Find a numerical value of $e^{2(\cos(\sqrt{2})+\sin(5))} + \ln 2$. Be careful to use square brackets and parentheses properly.

3 Define and graph functions

To define a function, we use a combination of the underscore sign "_" and the assignment operator ":=" as follows.

- (15) f[x_] := Sin[x]+Cos[x] (notice the underscore after x) followed by Shift+Enter. Then type
 f[Pi]+f[Pi/4] followed by Shift+Enter. Can you find a numerical value of this number?
- (16) Repeat the previous exercise but now replace the name f by a different name (say g) and drop the underscore after x. What do you learn about the role of the underscore sign?

Note that the underscore is required only when *defining* a function. It is not needed when *calling* the function.

- (17) Define two functions $f(x) = \ln x$ and $g(x) = \tan(\cos x)$. Then evaluate numerically the quantities f(f(10)) and $f\left(1 + g\left(\frac{\pi}{8}\right)\right)$ with 4 significant digits.
- (18) To clear a quantity from the memory, use the command Clear. Type Clear[f] followed by Shift+Enter. Then type f[Pi]+f[Pi/4] followed by Shift+Enter. What do you observe?
- (19) Define the function f(x) = sin(xe^x) and then evaluate f(e²) with 8 significant digits.
 Warning: you can type the product xe^x as either x*E^x or x E^x (space between x and E^x). If you type xE^x, Mathematica will understand xE as the name of a single variable.

Now let us plot functions of one variable. Try the following command:

- (20) $Plot[Sin[x]+Cos[2x], \{x,0,2*Pi\}], then Shift+Enter.$
- (21) To customize the style of the graph, you can add the option PlotStyle to the Plot command. Try the following:

 $Plot[Sin[x]+Cos[2x], \{x,0,2*Pi\}, PlotStyle \rightarrow \{Red, Dashed\}]$

Then Shift+Enter. Note that the arrow is typed as ->.

(22) You can also give the function a name before plotting it. For example,

f[x_] := Sin[x]+Cos[2x];
Plot[f[x], {x,0,2*Pi}, Filling -> Axis]

Then Shift+Enter. In the second command, note that the underscore within the brackets is no longer needed because f was already defined.

(23) To graph two functions, say sin(x) and cos(x), on the same plot, we do the following:

Plot[{Sin[x],Cos[x]},{x,0,5*Pi}]

To graph sin(x) with dashed line with green color, and graph cos(x) with thin line with red color, do the following:

Plot[{Sin[x],Cos[x]},{x,0,5*Pi}, PlotStyle->{{Dashed,Green},{Thin,Red}}]

To label the graphs, add the option PlotLegends->Automatic into the Plot command.

- (24) Graph the functions x, x^2, x^3, x^4, x^5 on the interval $x \in [0, 1]$ on the same plot. Describe the behavior of x^n as n increases.
- A piecewise function is a function given by different formulas for each given interval. For example,

$$f(x) = \begin{cases} 2x & \text{if } x \ge 0\\ x^2 & \text{if } x < 0 \end{cases}, \quad g(x) = \begin{cases} \sin x & \text{if } x < -1\\ e^x & \text{if } -1 \le x \le 1\\ \cos x & \text{if } x > 1 \end{cases}$$

are piecewise functions. We use the command **Piecewise** to define a piecewise function. The syntax is as follows:

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Piecewise[\{\{val_1, cond_1\}, \{val_2, cond_2\}, \ldots\}]
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represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

(25) Use the following commands to define the function f(x) above and plot it:

Make sure that you don't miss the underscore after x. Press Enter to go from the first line to the second line. Then press Shift+Enter to execute the entire block.

(26) Define and graph the function g given above.

4 Find derivative of a function

(27) To find the first, second, and third order derivatives of the function $\cos(x)$, try the following:

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D[Cos[x],x]
D[Cos[x],{x,2}]
D[Cos[x],{x,3}]
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Define the function $f[x_]:=Cos[x]$ and repeat the above commands but now replace Cos[x] by f[x].

- (28) Alternatively, you can simply run f'[x], f''[x], f'''[x].
- (29) To evaluate $f''(-\pi/4)$, run the command f'' [-Pi/4]. Alternatively, you can run

D[f[x], {x, 2}] /. {x -> -Pi/4}

Here, the symbol "/." is called the *replacement* (or *substitution*) operator. The above command will first differentiate the function f(x) twice, and then substitute $x = -\pi/4$.

- (30) Find the third derivative of the function $f(x) = e^{\cos(x^2)}$. Then evaluate numerically f''(1) using 6 significant digits.
- (31) Define the piecewise function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0\\ 0 & \text{if } x = 0 \end{cases}$$

In the **Piecewise** command, the unequal sign \neq is typed as != and the equal sign = is typed as ==. Find f'(x).

You will see that f'(0) = 0 according to Mathematica. Mathematica's answer that f'(0) = 0 is actually incorrect! One cannot differentiate a formula that is valid at only a single point; it is also necessary to understand how the function behaves in a neighborhood of this point. This example shows that you shouldn't blindly believe Mathematica although it is quite reliable most of the time.