Lab 2

In this lab, we will practice the following topics on Mathematica:

- Find indefinite integral (antiderivative)
- Find definite integral
- Find area of region between two curves
- Factor and expand a polynomial
- Find quotient and remainder of a polynomial division
- Find partial fraction decomposition of a rational function

The next lab will be built upon this lab, so please make sure that you go through all the instruction carefully and do all the assignments.

1 To turn in

Do Problems 1-33 in a single Mathematica Notebook file, or ipynb file if you use JupyterLab. Write *your name* and *lab number* at the beginning of your report. Clearly label each problem to separate them from other problems. Make sure to comment on each problem. If your code doesn't run correctly, explain what you are trying to do. **Failed code without any comment/explanation will receive 0 point.** Submit on Canvas both the *pdf file* and the *source file* (nb or ipynb).

Problems	Points
1, 6-8, 13-15, 18, 20, 21, 25, 27, 29	1
2-5, 9-12, 19, 23, 15, 19, 23, 30-33	2
16, 17, 22, 24, 26, 28	3
Readability of your report	3
Total: 33	Total: 62

2 Find an antiderivative

Function $F(x)$ is an *antiderivative* of $f(x)$ if $F' = f$. The command to find antiderivative of a function is **Integrate**.

(1) For example, try the following to get an antiderivative of x^2.

```
Integrate[x^2,x]
```

(2) Define the function

$$f(x) = \begin{cases} 2xe^x & \text{if} \quad x \geq 0 \\ x^2 & \text{if} \quad x < 0 \end{cases}$$

using **Piecewise** and find an antiderivative. Call this function $F(x)$.

(3) Graph $f(x)$ and $F(x)$ on the interval $[-2, 2]$ on the same plot.

(4) Do Exercise 4 of Section 7.3 in the textbook.

(5) Do Exercise 14 of Section 7.3 in the textbook.

3 Evaluate definite integrals

To evaluate the exact value of a definite integral $\int_a^b f(x)dx$, we use the command **Integrate** with the syntax

```
Integrate[f[x],{x,a,b}]
```

Sometimes, it is impossible to get the exact value of an integral. In that case, Mathematica only returns the input as is after taking a long time of processing. If you are using the Wolfram Cloud, you should terminate the execution by pressing the combination Alt+. if Mathematica takes too long to return a value. In that case, wrap the command **Integrate** inside the **N[...]** to get an approximate numerical value of the definite integral.

(6) Find the exact value of the integral $\int_1^4 \frac{1}{x}dx$.

(7) Try to evaluate the exact value of the integral $\int_0^2 e^{x^2}dx$ using **Integrate**. Does it give you a value?

(8) Now try `N[Integrate[...]]` instead of `Integrate`:

```
N[Integrate[E^(x^2), {x,0,2}], 8]
```

(9) Find the exact value of $\int_0^{\pi^2} \cos(\sqrt{x})dx$.

(10) Evaluate the integral $\int_0^{\pi^2} \cos(\sqrt{x} - x)dx$ correct to 10 digits after the decimal point.

(11) Do Exercise 10 of Section 7.2 in the textbook.

(12) Do Exercise 32 of Section 7.2 in the textbook.

4 Find area of region between two curves

In Math 212 (Calculus I), you learned that the area of the region bounded by the curves $y = f(x)$, $y = g(x)$, $x = a$, $x = b$ is

$$\int_a^b |f(x) - g(x)|dx$$

It is useful to shade the region between curves on Mathematica, especially when you are asked to find its area. Instruction manuals and textbooks are full of pictures of this kind.

(13) To shade the region between two curves, we use the option **Filling**. Try the following:

```
Plot[{Sin[x], Cos[x]}, {x, 0, 10}, Filling -> {1 -> {2}}]
```

In the above command, number 1 refers to the first function on the list (the sine function). Number 2 refers to the second function on the list (the cosine function). The part `1->{2}` means that we fill the region between curve 1 and curve 2 with the color of curve 1.

(14) You can shade the same region by the color of curve 2 by changing `1->{2}` to `2->{1}`. Try it.

(15) If you want to shade the region by a color of your choice, say Green, then change `1->{2}` to `1->{{2},Green}`. Try it. Then try `LightGreen` instead of `Green`.

(16) Execute the commands below and explain what each command does.

```
p1 = Plot[{Sin[x], 0}, {x, 0, Pi/2}, Filling -> {1 -> {2}}]
p2 = Plot[{1, 0}, {x, Pi/2, 2}, Filling -> {1 -> {2}}]
Show[p1, p2, PlotRange -> All]
```

(17) Shade the region bounded by the curves $y = \sqrt{x}$, $x + y = 2$, and the x-axis. Then find its area. Recall that $\sqrt{x}$ is typed as `Sqrt[x]`.

5 Factor and expand a polynomial

(18) Type

```
Expand[(1+x)^2]
Expand[(1+x)^3]
Expand[(a+2b)^4]
```

(press `Enter` to go to from one line to the next) then press `Shift+Enter` to execute the block. What does the command `Expand` do?

(19) Expand the polynomial $(1 + x)^5(2 - x)^4$.

(20) Now try the command

```
TraditionalForm[Expand[(1+x)^5*(2-x)^4]]
```

What difference do you see in the output compared to the output of the previous command?

(21) The syntax `func[expr]` is equivalent to `expr // func`. The second way does not need the square brackets and is sometimes more convenient than the first way. Try the following.

```
(1+x)^5*(2-x)^4 // Expand // TraditionalForm
```

(22) Expand the polynomial $f(x) = (2x^2 + x + 1)^7(x - 1)^3$ and arrange the terms in descending powers. What is the degree of f ?

(23) Try the commands

```
Factor[x^3+x^2-2]
x^3+x^2-2//Factor
x^3+x^2-2//Factor//TraditionalForm
```

What does the command `Factor` do?

(24) Factor the following polynomial and find all the real roots together with their multiplicities

$$f(x) = x^7 - 6x^6 + 11x^5 - 22x^3 + 20x^2 + 8x - 16$$

(25) To simplify the rational function $\dfrac{x^3 + x^2 - 2}{x^2 - 3x + 2}$, try the following commands

```
Simplify[(x^3+x^2-2)/(x^2-3x+2)]
Simplify[(x^3+x^2-2)/(x^2-3x+2)] // TraditionalForm
```

(26) Simplify the function $\cos^4(3x) - \sin^4(3x)$ and $\cos^4(3x) + \sin^4(3x)$. Note: the function $\cos^4(3x)$ is typed in as `Cos[3x]^4`.

6 Find the quotient and remainder of a polynomial division

Let $f(x)$ and $g(x)$ be two polynomials. The rational function $\frac{f(x)}{g(x)}$ can be written as

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

where $q(x)$ is the quotient and $r(x)$ is the remainder whose degree is less than the degree of g. Normally, you find $q(x)$ and $r(x)$ by hand using long division. On Mathematica, you can find $q(x)$ and $r(x)$ via the command **PolynomialQuotientRemainder**. The syntax is

```
PolynomialQuotientRemainder[f(x),g(x),x]
```

The output is `{q(x),r(x)}`.

(27) Let us consider the division $(x^4 + 2x + 1) \div (x^2 + 1)$. Try the following:

```
PolynomialQuotientRemainder[x^4 + 2x + 1, x^2 + 1, x]
PolynomialQuotientRemainder[x^4 + 2x + 1, x^2 + 1, x]//TraditionalForm
```

What are the dividend, divisor, quotient, and remainder of this division?

(28) Find the quotient and remainder of the division $(x^6 - 1) \div (x - 2)$. Then use this result to find the integral

$$\int \frac{x^6 - 1}{x - 2} dx$$

without using the command **Integrate**. Please type out your answer.

7 Partial fraction decomposition

The command `Apart` decomposes a rational function into simple fractions.

(29) For example, the decompose the function

$$f(x) = \frac{x^4 - 1}{(x - 2)(x - 3)}$$

into partial fractions, try the following:

```
Apart[(x^4-1)/((x-2)*(x-3))]
Apart[(x^4-1)/((x-2)*(x-3))]//TraditionalForm
```

(30) Decompose the function

$$\frac{x^4 + 1}{x^5 + 4x^3}$$

into partial fractions.

(31) Decompose the function

$$\frac{1}{(x^2 - 9)^2}$$

into partial fractions.

(32) Do Exercise 3b of Section 7.4 in the textbook.

(33) Do Exercise 6a of Section 7.4 in the textbook.