Lecture 1

Monday, January 13, 2025 10:09 AM

Objective: integration by parts

Motivation: to replace a difficult integral $\int f(x)dx$ with an equivalent but easier integral $\int g(x)dx$. The substitution method we learned in Calculus I was a method to do so. Recall
$\int g\big(u(x)\big)u'(x)dx = \int g(u)du$
The substitution method is based on the chain rule of differentiation. Now we
Now we learn another method, called *integration by parts*. This method is based on the product rule of differentiation. Recall: $(uv)' = uv' + u'v$
Integrate both sides:

$$uv = \int uv'dx + \int u'vdx$$

Thus,

$$\int uv'dx = uv - \int u'vdx$$

Imagine that the left hand side is a difficult integral that we need to find, and that the integral on the right hand side is easier to find. Of course, it is not always the case.

Ex: $\int xe^x dx$
The choice $u = e^x, v' = x$ will make the problem more difficult, not easier.
The choice $u = x, v' = e^x$ will make the problem easier. Indeed, $u' = 1$ and $v = e^x$. So,
$\int xe^x dx = \int uv'dx = uv - \int u'vdx = xe^x - \int e^x dx = xe^x - e^x + C$
Double check by differentiation.

Ex: $\int x \sin x \, dx$
Choose $u = x$ and $v' = sin\, x$.

Ex: $\int x^2 \cos x \, dx$
We will need to do integration by parts twice.