

Lecture 10

Tuesday, January 28, 2025 1:42 AM

What if $\deg P \geq \deg Q$? In this case, you need to do *long division* to write the fraction $P(x)/Q(x)$ in the form

$$\frac{P(x)}{Q(x)} = g(x) + \frac{r(x)}{Q(x)}$$

where $g(x)$ is the quotient and $r(x)$ is the remainder. Because $\deg r < \deg Q$, you can find the integral of $r(x)/Q(x)$ using the methods we discussed before.

Example:

$$\int \frac{x^3 + x}{x - 1} dx$$

Example:

$$\int \frac{x^4 + x^2 + 1}{x^2(x + 1)} dx$$

Example:

$$\int \sec^3 x \, dx = \int \frac{1}{\cos^3 x} \, dx = \int \frac{\cos x}{\cos^4 x} \, dx = \int \frac{\cos x}{(1 - \sin^2 x)^2} \, dx = \int \frac{du}{(1 - u^2)^2}$$

Partial fraction decomposition:

$$\frac{1}{(1 - u^2)^2} = \frac{1}{(u - 1)^2(u + 1)^2} = \frac{A}{u - 1} + \frac{B}{(u - 1)^2} + \frac{C}{u + 1} + \frac{D}{(u + 1)^2}$$