Lecture 11

Wednesday, January 29, 2025 11:48 AM

You can evaluate a definite integral numerically using Riemann sum:

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i^*)\frac{b-a}{n} = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i^*)$$

where $x_i^* \in [x_{i-1}, x_i]$.

- Left-point Riemann sum: $x_i^* = x_{i-1}$
  $$L_n = \frac{b-a}{n}\sum_{i=1}^{n} f(x_{i-1})$$
- Right-point Riemann sum: $x_i^* = x_i$
  $$R_n = \frac{b-a}{n}\sum_{i=1}^{n} f(x_i)$$
- Midpoint Riemann sum: $x_i^* = \frac{x_{i-1}+x_i}{2}$
  $$M_n = \frac{b-a}{n}\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)$$
- Trapezoid sum: approximate the area under the curve and above the interval $[x_{i-1}, x_i]$ by the area of the trapezoid
  $$\int_{x_{i-1}}^{x_i} f(x)\,dx \approx \frac{f(x_{i-1})+f(x_i)}{2}\frac{b-a}{n}$$
  Thus,
  $$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n}\frac{f(x_{i-1})+f(x_i)}{2}\frac{b-a}{n} = \frac{b-a}{n}\sum_{i=1}^{n}\frac{f(x_{i-1})+f(x_i)}{2}$$

  $$T_n = \frac{b-a}{n}\sum_{i=1}^{n}\frac{f(x_{i-1})+f(x_i)}{2}$$

Example:
Approximate $\int_1^3 x^2 dx$ using $n = 4$.

(a) Trapezoid
(b) Midpoint
(c) Left-point
(d) Right-point