Lecture 13

Thursday, January 30, 2025 1:24 PM

Recall that we defined the definite integral $\int_a^b f(x)dx$ when
- a and b are numbers (not $-\infty$ or ∞),
- f is continuous on $[a, b]$.

Improper integral refers to an interpretation of $\int_a^b f(x)dx$ when one of these requirements is not satisfied. Improper integrals are defined based on Riemann integrals (but are not Riemann integrals themselves).

$$\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

$$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$$

$\int_a^b f(x)dx = \lim_{c\to b^-} \int_a^c f(x)dx$ if f is continuous on $[a, b)$ but not at b

$\int_a^b f(x)dx = \lim_{c\to a^+} \int_c^b f(x)dx$ if f is continuous on $(a, b]$ but not at a

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ if f is continuous on $[a, c) \cup (c, b]$ but not at c

An improper integral may converge (i.e. the limit exists as a number) or diverge.

Examples:

$$\int_0^\infty \frac{1}{x^2+1}dx$$

$$\int_1^\infty \frac{1}{x^2}dx$$

$$\int_0^1 \frac{1}{x^2}dx$$

Observations:
- $\int_0^1 \frac{1}{x^p}dx$ converges if and only if $p < 1$.
- $\int_1^\infty \frac{1}{x^p}dx$ converges if and only if $p > 1$.