Lecture 14

Monday, February 3, 2025 2:10 PM

Examples of improper integral:

$$\int_0^\infty e^{-x} dx$$

$$\int_1^\infty \frac{1}{x^2} dx$$

$$\int_0^1 \frac{1}{x^2} dx$$

$$\int_0^\infty \frac{1}{x^2} dx$$

Observations: for any $a > 0$,

- $\int_0^a \frac{1}{x^p} dx$ converges if and only if $p < 1$.
- $\int_a^\infty \frac{1}{x^p} dx$ converges if and only if $p > 1$.

Comparison Principle (the Limit version):
Suppose $f(x), g(x) \geq 0$ for all $x \geq a$.

- If $\int_a^\infty g(x)dx$ converges and $\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty$ then $\int_a^\infty f(x)dx$ also converges.
- If $\int_a^\infty g(x)dx$ diverges and $\lim_{x\to\infty} \frac{f(x)}{g(x)} > 0$ then $\int_a^\infty f(x)dx$ also diverges.

Example 1:

$$\int_3^\infty \frac{1}{x^2 - 4} dx$$

$f(x) = \frac{1}{x^2-4}$ and $g(x) = \frac{1}{x^2}$

Example 2:

$$\int_{-\infty}^{-1} \frac{e^x}{x} dx$$