Lecture 17

Monday, February 10, 2025   1:18 AM

We are now in Chapter 9: Differential equations.
Differential equations are everywhere in real life. It is a mathematical language to describe all kind of motions of life.
A differential equation is an equation that involves a function, its variable, and one or more of its derivatives.
Example:
$y' = y$
$y' = x + y$
$yy'y'' = xe^y$
It is important to note that these equations have to be satisfied for all x. The unknown is not x, but y as a function of x.

Example:
Pendulum motion:
$$\theta'' + \frac{g}{l}\sin\theta = 0$$
Water level in a draining tank: $h = h(t)$ is the depth difference between the water level and the (small) opening where water comes out. In most cases, the water is incompressible, which means that the volume that leaves must equal the volume that is lost. Let A be the area of the water surface and a be the are of the opening.
Then $V_{drop} = -A\Delta h$ and $V_{leave} = a\Delta s \approx av\Delta t$. Torricelli's Law states that $v = \sqrt{2gh}$ where g is the graviational constant. Then
$$-A\Delta h \approx av\Delta t = a\sqrt{2gh}\Delta t$$
Divide both sides by Δt:
$$\frac{\Delta h}{\Delta t} \approx -\frac{a}{A}\sqrt{2gh}$$
Taking $\Delta t \to 0$, we obtain a differential equation:
$$h' = -\frac{a}{A}\sqrt{2gh}$$

Work on the worksheet.