Lecture 22

Wednesday, February 19, 2025 1:30 AM

The separation of variables method and the integrating factor method can help you find an analytic form of the solution. However, they are quite sensitive to the form of the equation. For example,

- $y' = \sin y$ can be solved by separation of variables, but $y' = x + \sin y$ can't.
- $y' = xy$ can be solved by both methods, but $y' = xy + y^2$ can't be solved by either method.

There is a very intuitive method, the *Euler's method*, that can help you solve any differential equation of the form $y' = f(x, y)$ where f is a given function. However, it only solve the equation *approximately*. Let's consider the following example:

$$y' = x + y^2, \qquad y(0) = 1$$

The unknown is the function $y = y(x)$. Geometrically, you may think of a function as a curve (its graph). What are the two equations above inform you about the curve? The equation $y(0) = 1$ tells you that the curve must pass by the point $(0,1)$. Imagine that you are walking on the curve. Your initial position is at $(0,1)$. The equation $y' = x + y^2$ tells you that whenever you are at a position (a, b), your next step has a slope of $y'(a) = a + b^2$.

Principle: if you know where you are (current position), you know what direction to take the next step.