Lecture 25

Tuesday, February 25, 2025   3:06 AM

According to Malthus' model, the population grows exponentially in time. It has a good predicting power over a short period of time. Over a long time, Malthus' model is no longer reasonable.

In 1838, Pierre Verhulst published a paper on an alternative model which he termed the *logistic model*. This model prevents the population from growing exponentially. It suggests that there is a sustainable population M in the sense that: over a long period of time, the population will approach M, no matter what the initial population is (as long as it is positive). Verhulst's model seems more realistic than Malthus' model. It is summarized in the following differential equation:

$$y' = ry\left(1 - \frac{y}{M}\right)$$

Here, r is the birth rate and M is the carrying capacity (or sustainable population).
You can solve this equation using the separation of variable method, followed by partial fraction decomposition. You will finally arrive at

$$y = \frac{M}{1 + ce^{-rt}}$$

where c is a constant to be determined by the initial condition.