Lecture 30

Tuesday, March 4, 2025 10:10 PM

A *polar curve* is a curve described in polar coordinates (r, θ) by $r = r(\theta), a \leq \theta \leq b$.
Equivalently, it is a parametric curve in the Cartesian coordinates where

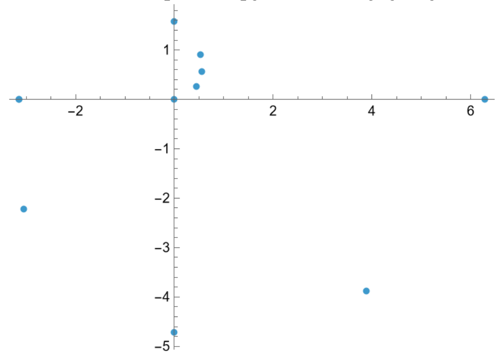
$$x = r\cos\theta = r(\theta)\cos\theta$$
$$y = r\sin\theta = r(\theta)\sin\theta$$

Graph the following polar curves:

1) $r = \theta$

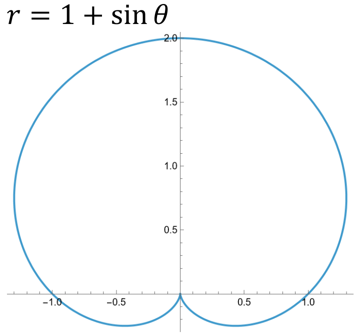
θ	r
0	0
$\pi/6$	$\pi/6$
$\pi/4$	$\pi/4$
$\pi/3$	$\pi/3$
$2\pi/3$	$2\pi/3$
π	π
$5\pi/4$	$5\pi/4$
$3\pi/2$	$3\pi/2$
$7\pi/4$	$7\pi/4$
2π	2π

ListPolarPlot[Table[{a*Pi,a*Pi},{a, {0,1, 1/6,1/4,1/3,1/2,1, 6/5,3/2,7/4,2}}]]

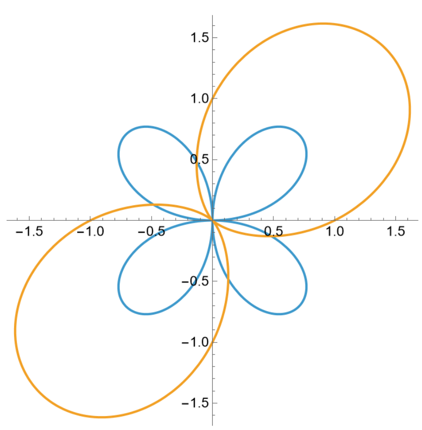


PolarPlot[t, {t,0,2 Pi}]

2) $r = 1 + \sin\theta$



Intersection between two curves $r = \sin 2\theta$ and $r = 1 + \sin 2\theta$



The intersection point can have more than one equivalent polar coordinates: (r_1, θ_1) and (r_2, θ_2).
We have $r_1 = \sin 2\theta_1$ and $r_2 = 1 + \sin 2\theta_2$.
For these polar coordinates to be equivalent, one of the three following scenarios must happen:

1) Either $r_1 = r_2$ and $\theta_1 = \theta_2 + k2\pi$
2) $r_1 = -r_2$ and $\theta_1 = \theta_2 + \pi + k2\pi$
3) $r_1 = r_2 = 0$ and θ_1, θ_2 are arbitrary

The first scenario cannot happen. For the second scenario to happen, we need $\sin 2\theta_1 = -1 - \sin 2\theta_1$, which implies $\sin 2\theta_1 = -1/2$. Thus, $2\theta_1 = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$. Then

$$\theta_1 = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

The last scenario happens when $\theta_1 = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{4}$.
These values of θ_1 correspond to 5 intersection points on the picture.