Lecture 33

Wednesday, March 12, 2025 1:22 AM

A sequence (a_n) is a list of numbers $a_1, a_2, a_3, \dots$ It is usually used as a means of approximation.

For example, what is the value of π? It is an irrational number, thus having no repeating pattern in the decimal form. The answer depends on the accuracy that you want. In the Bible (1 Kings 7:23), $\pi \approx 3$. In elementary school, you were taught $\pi \approx 3.14$. Later, you found out more and more digits of π. There is a sequence of numbers that approximate π. For example,

3, 3.1, 3.14, 3.145, 3.1459,…

This is a sequence with $a_n =$ the first n digits of π. It is not the only sequence that approximates π. For example,

$$4, 4\left(1-\tfrac{1}{3}\right), 4\left(1-\tfrac{1}{3}+\tfrac{1}{5}\right), 4\left(1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}\right), 4\left(1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{9}\right), \dots$$

is another sequence that approximates π. The above formula was found by James Gregory in 1671 and Gottfried Leibniz in 1673. The general term of the above sequence is:

$$b_n = 4\sum_{k=1}^{n} \frac{(-1)^{k-1}}{2k-1}$$

Notations: $\{a_n\}$, $\{a_n\}_{n=3}^{\infty}$, $\{a_n\}_{n\geq 3}$, (a_n), $(a_n)_{n=3}^{\infty}$, $(a_n)_{n\geq 3}$

Some exercises on listing a sequence given a formula for the general term or a recursive formula (see worksheet).
Some exercises on finding the general term based on the first few terms.

Increasing sequence: $a_{n+1} \geq a_n$ for every index n
Decreasing sequence: $a_{n+1} \leq a_n$ for every index n

Example: the sequence $a_n = \frac{2^n}{2n-1}$ is increasing for $n \geq 2$ but not for $n \geq 1$.
How to prove? Note that $a_n > 0$. Show that either $a_{n+1} - a_n < 0$ or $\frac{a_{n+1}}{a_n} < 1$.