Lecture 34

Thursday, March 13, 2025 2:54 PM

A sequence is said to be *monotone* if it is increasing or decreasing.
Example: show that the sequence $a_n = 2^n/n$ is increasing.
Two common methods:

- Show that $a_{n+1} - a_n \geq 0$ for all n
- Show that $\frac{a_{n+1}}{a_n} \geq 1$ for all n

Note that listing terms of the sequence and see the increasing/decreasing trend doesn't constitute a proof. There is no guarantee that this trend will continue forever. You must use a valid argument to establish the fact that $a_{n+1} \geq a_n$ (or $a_{n+1} \leq a_n$) for all n.

Work on an example on the worksheet.

A sequence is bounded from above by M if $a_n \leq M$ for all index n.
A sequence is bounded from below by m if $a_n \geq m$ for all index n.

Important facts:
If a sequence is increasing and bounded from above, then it must have a limit.
If a sequence is decreasing and bounded from below, then it must have a limit.