Lecture 36

Monday, March 17, 2025 9:42 AM

There is a theorem that relates monotonicity, boundedness, and limit of a sequence.

Theorem:

- If a sequence is increasing and bounded from above then it has a limit.
- If a sequence is decreasing and bounded from below then it has a limit.

L'Hospital rule:

If f and g are differentiable functions and the limit $\lim_{x\to a}\frac{f(x)}{g(x)}$ is in an indefinite form ($\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$) then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ if the latter exists.

Examples:

1) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} \overset{L'H}{=} \lim\limits_{x\to 0}\dfrac{e^x}{1} = e^0 = 1$
2) $\lim_{x\to 0}\frac{e^x-1}{x+1} \overset{L'H}{=} \lim_{x\to 0}\frac{e^x}{1} = e^0 = 1$ (wrong!)
   It is wrong because the limit is not in an indefinite form. You can just plug $x = 0$ to the function to get the limit.
   $\lim\limits_{x\to 0}\dfrac{e^x-1}{x+1} = \dfrac{e^0-1}{0+1} = 0$
3) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2} \overset{L'H}{=} \lim\limits_{x\to 0}\dfrac{\sin x}{2x} \overset{L'H}{=} \lim\limits_{x\to 0}\dfrac{\cos x}{2} = \dfrac{\cos 0}{2} = \dfrac{1}{2}$
4) Find the limit of the sequence $a_n = n\sin\frac{1}{n}$
   $\lim a_n = \lim\limits_{x\to\infty} x\sin\dfrac{1}{x} = \lim\limits_{x\to\infty}\dfrac{\sin(1/x)}{1/x} = \lim\limits_{t\to 0}\dfrac{\sin t}{t} \overset{L'H}{=} \lim\limits_{t\to 0}\dfrac{\cos t}{1} = \cos 0 = 1$

Work on Problems 8, 9, 10 on the last worksheet.