Lecture 37

Tuesday, March 18, 2025 2:50 AM

A series is a sum of infinitely many terms: $\sum_{n=1}^{\infty} a_n$
Here, a_n is the general term of the series. Each sequence $\{a_n\}$ is associated with a series. The index n in the sum can be renamed without changing the sum:

$$\sum_{n=1}^{\infty} a_n = \sum_{k=1}^{\infty} a_k = \sum_{l=1}^{\infty} a_l = \cdots$$

To be more precise,

$$\sum_{n=1}^{\infty} a_n = \lim_{m\to\infty} \sum_{n=1}^{m} a_n = \lim_{m\to\infty} s_m$$

The sum $s_m = a_1 + \cdots + a_m$ is called the m'th partial sum.
A series is said to converge if the sequence s_m converges (to a number). Otherwise, it is said to diverge.

Example:
Find the 3rd and 5th partial sum of the series $\sum_{n=1}^{\infty} n$
Does the series converge? If so, what is the value of the series?

Example:

$$\sum_{n=1}^{\infty} 1 = ?$$
$$\lim_{n\to\infty} 1 = ?$$

The series is defined as the limit of the partial sums, not as the limit of the associated sequence.

Series is often an effective calculation tool. For example,

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right) = 4\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$$
$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}$$

The most common/interesting question about a series is whether it converges. The second most common/interesting question is to calculate the value of the series (if it converges).

In general, the first question is much easier to answer than the second question. In some situations, it is easy to answer both. That is the case with the *geometric series* and the *telescoping series*.

Geometric series: $\sum a_n$ where the quotient a_{n+1}/a_n is a constant.
Example:
$1 + 2 + 4 + 8 + 16 + \cdots$
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$
$3 + 6 + 9 + 12 + 15 + \cdots$