Lecture 39

Thursday, March 20, 2025 5:35 AM

Note on limit of a sequence:
$\{a_n\}$: $a_1, a_2, a_3, a_4, \ldots$
$\{a_{n+1}\}$: $a_2, a_3, a_4, \ldots$
$\{a_{n+100}\}$: $a_{100}, a_{101}, a_{102}, \ldots$
All of these sequences have the same limit. The limit of a sequence doesn't depend on the "head" of the sequence, but the "tail" of it.

In the same spirit, the *convergence* of the series doesn't depend on the head of the series but on the tail of it. (However, the *value* of the series depends on both head and tail.) For this reason, when examining the convergence of the series, you can be vague about the starting index of the series by writing $\sum a_n$. This style of writing will be used for the next couple weeks as you learn how to know if a series converges or diverges.

Example:

$$2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\frac{2}{81}+\cdots=\frac{2}{1-\frac{1}{3}}=3$$

$$1+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\frac{2}{81}+\cdots=2$$

Divergence Test:
If the series $\sum a_n$ converges then $\lim a_n=\lim(s_n-s_{n-1})=\lim s_n-\lim s_{n-1}=0$.
Therefore, if $\lim a_n \neq 0$ then the series $\sum a_n$ doesn't converge.
Example:

$$\sum n$$

$$\sum \frac{n}{n+1}$$

$$\sum (-1)^n \frac{n}{n+1}$$

$$\sum \frac{1}{n}$$

The last series is inconclusive from the Divergence Test. You will see later that it diverges.