Lecture 40

Friday, March 21, 2025 1:44 PM

Integral Test:
If $a_n = f(n)$ and f is an decreasing and nonnegative function on an interval $[0, M]$ then $\sum a_n$ converges if and only if $\int_M^\infty f(x)dx < \infty$.

Why?
Since each $a_n \geq 0$, we need to explain why $\sum a_n < \infty$. For $n > M$,

$$a_n = f(n) = \int_{n-1}^{n} f(n)dx \leq \int_{n-1}^{n} f(x)dx$$

Sum over all $n > M$

$$\sum_{M+1}^{\infty} a_n \leq \sum_{M+1}^{\infty} \int_{n-1}^{n} f(x)dx = \int_M^\infty f(x)dx < \infty$$

Examples:
p-series $\sum \frac{1}{n^p}$

$$\sum \frac{1}{n^2+1}$$

$$\sum \frac{1}{\ln n}$$

$$\sum \frac{1}{n^3 - n}$$