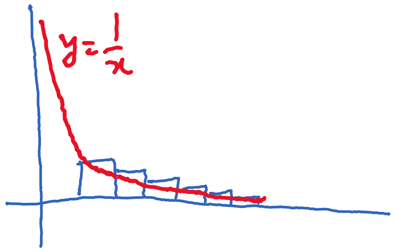
Lecture 41

Monday, March 24, 2025 1:11 PM

Why is $\sum 1/n = \infty$?

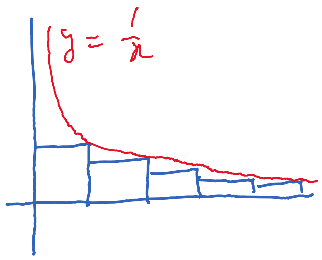


$\sum_{n=2}^{\infty} \frac{1}{n}$ = sum of areas of rectangles

$\geq$ area under the curve $y = \frac{1}{x}$ where $x \in [1, \infty)$

$= \int_1^{\infty} \frac{1}{x}\,dx = \infty$

Why doesn't the following picture help?



$\sum_{n=2}^{\infty} \frac{1}{n} \leq$ area under the curve $y = \frac{1}{x}$

$= \int_1^{\infty} \frac{1}{x}\,dx = \infty$

You can't conclude whether $\sum \frac{1}{n} = \infty$ or finite.

Goal for today: Comparison test

Motivating example: $\sum \frac{1}{n^2+4}$

Method 1: Integral test

Method 2: compare $\frac{1}{n^2+4}$ with $\frac{1}{n^2}$