Lecture 42

Tuesday, March 25, 2025 4:09 AM

The Comparison principle has several variations. Analogy: if you score equally or better than your classmate in every assignment and he passes the class, then you should also pass the class.

Comparison principle 1: Assume that $0 \leq a_n \leq b_n$ for all n.

- If $\sum b_n$ converges then so does $\sum a_n$.
- If $\sum a_n$ diverges then so does $\sum b_n$.

Examples:

$$\sum \frac{1}{n2^n}, \sum \frac{1}{n(n^2+1)}, \sum \frac{1}{n+\sqrt{n}}$$

The last one may be a little tricky. Note that

$$\frac{1}{n+\sqrt{n}} \geq \frac{1}{n+n} = \frac{1}{2}\frac{1}{n}$$

Then

$$\sum \frac{1}{n+\sqrt{n}} \geq \frac{1}{2}\sum \frac{1}{n} = \infty$$

Comparison principle 2: Assume $|a_n| \leq b_n$ for all n. If $\sum b_n$ converges then so does $\sum a_n$.

Example:

$$\sum \frac{\sin n}{n^2}$$

Comparison principle 3:

- If $\sum b_n$ converges and $\lim \frac{a_n}{b_n}$ exists (not ∞) then $\sum a_n$ also converge.
- If $\sum b_n$ diverges, $b_n > 0$, and $\lim \frac{a_n}{b_n} > 0$ (possibly ∞) then $\sum a_n$ also diverge.

Example:

$$\sum \frac{1}{n^2-4}$$