

# Lecture 43

Thursday, March 27, 2025 2:18 AM

*Alternating series* is a series of the form

$$\sum (-1)^n b_n = -b_1 + b_2 - b_3 + b_4 - \dots$$

or

$$\sum (-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + \dots$$

where  $b_n > 0$  for all  $n$ .

**Examples and non-examples:**

$$\sum (-1)^n$$
$$\sum \frac{(-1)^n}{n^2}$$

$$\sum \sin n$$

$$\sum (-1)^n \sin n$$

**Alternating Series Test** (for convergence):

If  $b_n$  is a decreasing sequence and  $\lim b_n = 0$  then the series converges.

*Visualization:* Imagine a pendulum swinging back and forth:

- At first, it swings wide — a big arc to the right (like adding  $b_1$ ),
- Then swings back to the left, but not as far (subtracting  $b_2$ ),
- Then right again, but less than before ( $+b_3$ ),
- Then left again, smaller swing ( $-b_4$ ),
- And so on...

Each swing is smaller than the last, and over time the pendulum comes to rest at a central point.

**Exercises:**

$$\sum (-1)^n \frac{n}{n+1}$$

$$\sum \frac{(-1)^n}{2^n}$$

$$\sum (-1)^n \frac{n^2}{2^n}$$

$$\sum (-1)^n (\sqrt{n+1} - \sqrt{n})$$