Lecture 46

Tuesday, April 1, 2025 12:28 AM

Ratio Test:

Let $L = \lim \left|\frac{a_{n+1}}{a_n}\right|$.

- If $L < 1$ then the series $\sum a_n$ is absolutely convergent.
- If $L > 1$ then the series $\sum a_n$ is divergent.

Root Test:

Let $L = \lim \sqrt[n]{|a_n|}$.

- If $L < 1$ then the series $\sum a_n$ is absolutely convergent.
- If $L > 1$ then the series $\sum a_n$ is divergent.

These tests can be explained by comparing the series $\sum a_n$ with the geometric series $\sum b_n = \sum L^n$.

$\sqrt[n]{|a_n|} \approx L$, so $|a_n| \approx L^n$, so $\sum |a_n| \approx \sum L^n$

The latter series diverges when $L = 1$, so why don't we conclude that $\sum a_n$ diverges when $L = 1$?

$L = 1$ lies at the border line between convergence and divergence. Although it is true that $\sqrt[n]{|a_n|} \approx L$, the difference between two sides is magnified when you raise both sides to power n. If L stays away from the border line ($L < 1$ or $L > 1$), you have room for accommodate this large difference. Right at the border line ($L = 1$), the series may easily fall on one side or the other.

Examples:

$$\sum \frac{n^2}{2^n}$$

$$\sum \frac{3^n}{n!}$$

$$\sum (-1)^n \left(\frac{n+1}{2n-3}\right)^n$$

$$\sum \left(\frac{\sin n}{n}\right)^n$$

Example:

Determine the convergence or divergence of the series $\sum \frac{(x+1)^n}{n^2}$ depending on x.

This series is known as a power series centered at -1.