Lecture 47

Wednesday, April 2, 2025 2:49 AM

Power series is a series of the form $\sum c_n(x-a)^n$.
It is called a power series centered at a.
The *interval of convergence* of this series is the set of all values of x such that the series converges.

Example:
Find the interval of convergence of the power series $\sum \frac{x^n}{n}$.
$x=0$ is in the interval of convergence because $\sum \frac{0^n}{n} = 0$ (converges).
$x=1$ is not in the interval of convergence because $\sum \frac{1^n}{n} = \sum \frac{1}{n}$ diverges.
$x=-1$ is in the interval of convergence because $\sum \frac{(-1)^n}{n}$ converges.
To find all the values of x for which the series converges, we will use the Ratio Test.

Example:
Find the interval of convergence of the power series

$$\sum \frac{2^n}{n+1} x^n$$

$$\sum (-1)^n (x+2)^n$$

$$\sum \frac{x^n}{n!}$$

Suppose $\lim \left|\frac{c_{n+1}}{c_n}\right|$ exists (possibly infinity), then the *radius of convergence* of this series is $R = \frac{1}{\lim\left|\frac{c_{n+1}}{c_n}\right|}$.