Lecture 48

Friday, April 4, 2025 4:03 AM

Discuss the power series given last time:

$$\sum (-1)^n (x+2)^n$$

$$\sum \frac{x^n}{n!}$$

Two more examples:

$$\sum n!\, x^n$$

$$\sum \frac{x^n}{n^n}$$

Theorem:
Suppose the following limit exists (possibly 0 or ∞):

$$R = \frac{1}{\lim\left|\frac{c_{n+1}}{c_n}\right|} = \lim\left|\frac{c_n}{c_{n+1}}\right|$$

Then the series $\sum c_n (x-x_0)^n$ converges on the interval $(x_0 - R, x_0 + R)$ and diverges on $(-\infty, x_0 - R) \cup (x_0 + R, \infty)$.

R in the above theorem is called the *radius of convergence* of the series.

A power series defines/represents a function on the interval of convergence. Sometimes, the power series can be computed exactly. For example,

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

This (geometric) power series has many applications and is worth memorizing. Note that although the right hand side is defined for any $x \in R\backslash\{0\}$, the left hand side is only defined for $x \in (-1,1)$. The equality is thus valid only for $x \in (-1,1)$.

Some applications:

$$1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$$

$$1 - x + x^2 - x^3 + \cdots = \frac{1}{1+x}$$

$$1 + 2x + 4x^2 + 8x^3 + \cdots = \frac{1}{1-2x}$$