

Lecture 6

Tuesday, January 21, 2025 9:36 AM

Example:

$$\int_0^2 x^2 \sqrt{4-x^2} dx$$

Let $x = 2 \sin u$. So, $u = \arcsin\left(\frac{x}{2}\right)$.

x	0	2
u	0	$\pi/2$

$$dx = x' du = 2 \cos u du$$

$$\begin{aligned} \int_0^2 x^2 \sqrt{4-x^2} dx &= \int_0^{\pi/2} 4 \sin^2 u \sqrt{4-4 \sin^2 u} 2 \cos u du = \\ &= \int_0^{\pi/2} 4 \sin^2 u \sqrt{4 \cos^2 u} 2 \cos u du \\ &= \int_0^{\pi/2} 4 \sin^2 u (2 \cos u)(2 \cos u) du \\ &= 16 \int_0^{\pi/2} \sin^2 u \cos^2 u du = 4 \int_0^{\pi/2} \sin^2 2u du \\ &= 4 \int_0^{\pi/2} \frac{1 - \cos 4u}{2} du = 2 \int_0^{\pi/2} (1 - \cos 4u) du \\ &= 2 \left(x - \frac{\sin 4u}{4} \right) \Big|_0^{\pi/2} = \pi \end{aligned}$$

Example: $\int_0^{\sqrt{3}} \sqrt{1+x^2} dx$

Use the substitution $x = \tan u$.

Example: $\int_0^1 \sqrt{4+9x^2} dx$

Use the substitution $u = 3x$ and then use the substitution $u = 2 \tan v$.

Example: $\int_3^5 x \sqrt{x^2-9} dx$

Use the substitution $u = x^2 - 9$

Example: $\int_3^{2\sqrt{3}} x^2 \sqrt{x^2-9} dx$

Use the substitution $x = \tan u$.