Final project for Math 390R

Policies:

Your final project has two components: written report and oral presentation. **Choose only ONE of the two projects given below.** The deadline to submit your written report on Canvas is at midnight on April 15. Your oral presentation will be from 11 AM – 1:50 PM on April 16.

For the written report, Google Colab will be the best environment to write your report because you can render math formula nicely and execute Python commands. Include the following components:

- Your analytical method:
  - How do you model the given problem into a math problem?
  - What method/strategy do you use to solve this math problem? Try to use methods that you learned in class. If you use other methods, you will not get full credit.
  - Write the details of every step to arrive at the solution.
- Your computational method: If you use computer programming to perform any computational work, write down the code and explain it. The lab instruction from the lab assignments during the course is a useful resource for you. If you use exclusively computational methods that were not covered in lectures or labs, you will not get full credit.
- References: if you use any sources other than the course material, such as Wikipedia, online articles, YouTube videos, cite them. If you use AI tools such as ChatGPT, explain how you use it and what part of the problem you use it for.

For the oral presentation, you have up to 15 minutes to present your report and 10 minutes to answer questions from the audience. You may use your written report to present without having to make a set of PowerPoint slides. You can only use the files that have already been turned in on Canvas for presentation. You are expected to sit in the audience through all presentations other than your own. To get full credit, you need to ask at least one question when sitting in the audience. If you ask an insightful question, you can earn up to 2 bonus points.

Written report	Explain how to model the problem	/5
	Outline the method/strategy to solve	/5
	Provide adequate analytical details	/4
	Explain the computational method/algorithm	/4
	Readability of the report	/2
		Total: /20
Oral presentation	Explain how to model the problem	/3
	Outline the method/strategy to solve	/3
	Explain some main details of the method, including computational method/algorithm	/4
	Answer questions	/3
	Ask questions	/2
	Bonus points for good questions	/0
		Total: /15

Project 1:

A company distributes milk to two retail stores (called A and B) from two distribution centers (called I and II). The cost of shipping from distribution center I to store A is 12 cents per gallon, and to store B is 15 cents per gallon. The cost of shipping from distribution center II to store A is 14 cents per gallon, and to store B is 17 cents per gallon. Distribution center I can deliver up to 600 gallons per day while distribution center II can deliver up to 800 gallons per day. Store A's daily demand is at least 350 gallons while that of store B is at least 400 gallons.

1) Find the amount of milk that should be distributed from each distribution center to each store that minimizes the total shipping cost.

2) Milk has a limited shelf life. If the shipping of a batch of Q_0 gallons of milk is delayed by t days then the amount acceptable for delivery is $Q(t)$, a decreasing function of t. Suppose the spoilage of milk is modeled by the differential equation

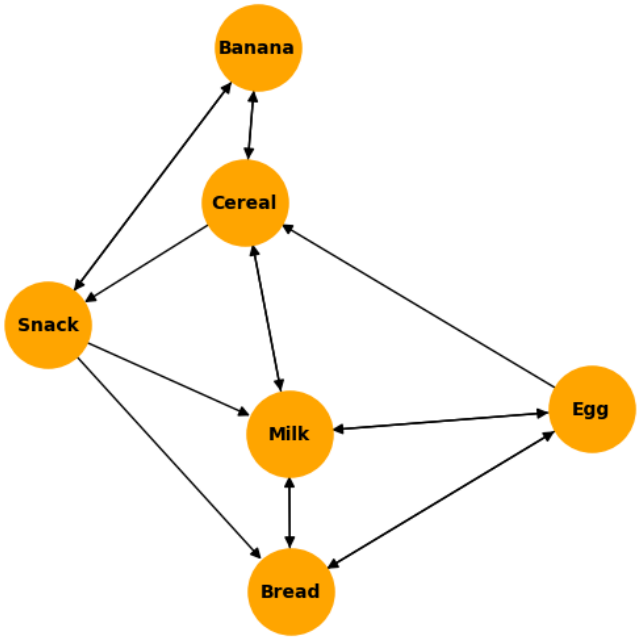
$$Q' = -\frac{0.08\, t\, Q^2}{20 + Q}$$

Distribution centers I and II currently hold 600 gallons and 800 gallons of milk, respectively. Suppose that they both have delays in shipping and are not restocked. Solve the optimization problem (as in Part 1) if the shipping delay is 1 day, 2 days, 3 days, 4 days, 5 days.

Project 2:

A retail store analyzes customer purchasing behavior and builds a *co-purchase* graph in which the nodes are the product categories (milk, egg, cereal, banana, bread, snack), while the arrow from A to B indicates that people who bought A usually also bought B. Co-purchase graph is used to identify how central or influential each product category is in allocating shelf space and aisle traffic in the store.

1) Consider the following co-purchase graph. Rank the product categories according to their influence.



2) The store manager has limited premium shelf space —such as areas near the entrance or at eye level—and wants to distribute this space among the product categories in a way that maximizes customer attention and sales potential. The problem is to determine the right proportion of premium shelf space allocated to each of them. The combined effect of customer attention and sales potential is modeled as a *utility function*

$$f(x,y) = \frac{p_A x}{1+x^2} + \frac{p_B y}{2+y^2}$$

where p_A and p_B are the PageRank of the two most influential products (called A and B), x the proportion of A, and y the proportion of B. Determine the proportions of premium shelf space for each product category that maximize the utility function.