Homework 5

1. Sketch by hand the contour map of the following functions.

   (a) $f(x, y) = x + y$

   (b) $f(x, y) = (x + y)^2$

   (c) $f(x, y) = x + y^2$

   (d) $f(x, y) = \sqrt{x}$

2. Explain how the contour maps of $f(x, y) = x + y$ and $g(x, y) = (x + y)^2$ are different from each other.

3. Match the following functions with their contour maps (Figure 1). Explain your answer.

   (a) $\frac{1}{x}$

   (b) xy^2

   (c) $y - \frac{1}{x}$

   (d) xy

4. Figure 2 is the contour map of a function. Determine approximately all local maximum values and local minimum values for $(x, y) \in [-3, 3] \times [-5, 5]$. At what positions (x, y) are these values attained? Explain your answer.

5. Find the partial derivatives of $f(x, y) = ye^{xy}$.

6. For each pair of vectors u and v below, do the following:

   - Plot vector u and v.
   - Find the length of u and v.
   - Find the dot product $u \cdot v$ and the angle (between $0°$ and $180°$) between them.

   (a) $u = (1, 2)$, $v = (2, 3)$

   (b) $u = (1, -1)$, $v = (2, 2)$

   (c) $u = (-2, 1)$, $v = (2, 1)$

7. Let $f(x, y) = x/y$.

   (a) Find the gradient of f.

   (b) Find the gradient vector of f at the point $P(3, 1)$.

   (c) Find the rate of change (directional derivative) of f in the direction of vector $u = (2, -1)$.

   (d) At the point $P(3, 1)$, in what direction does f have the maximum rate of change? What is this maximum rate of change?

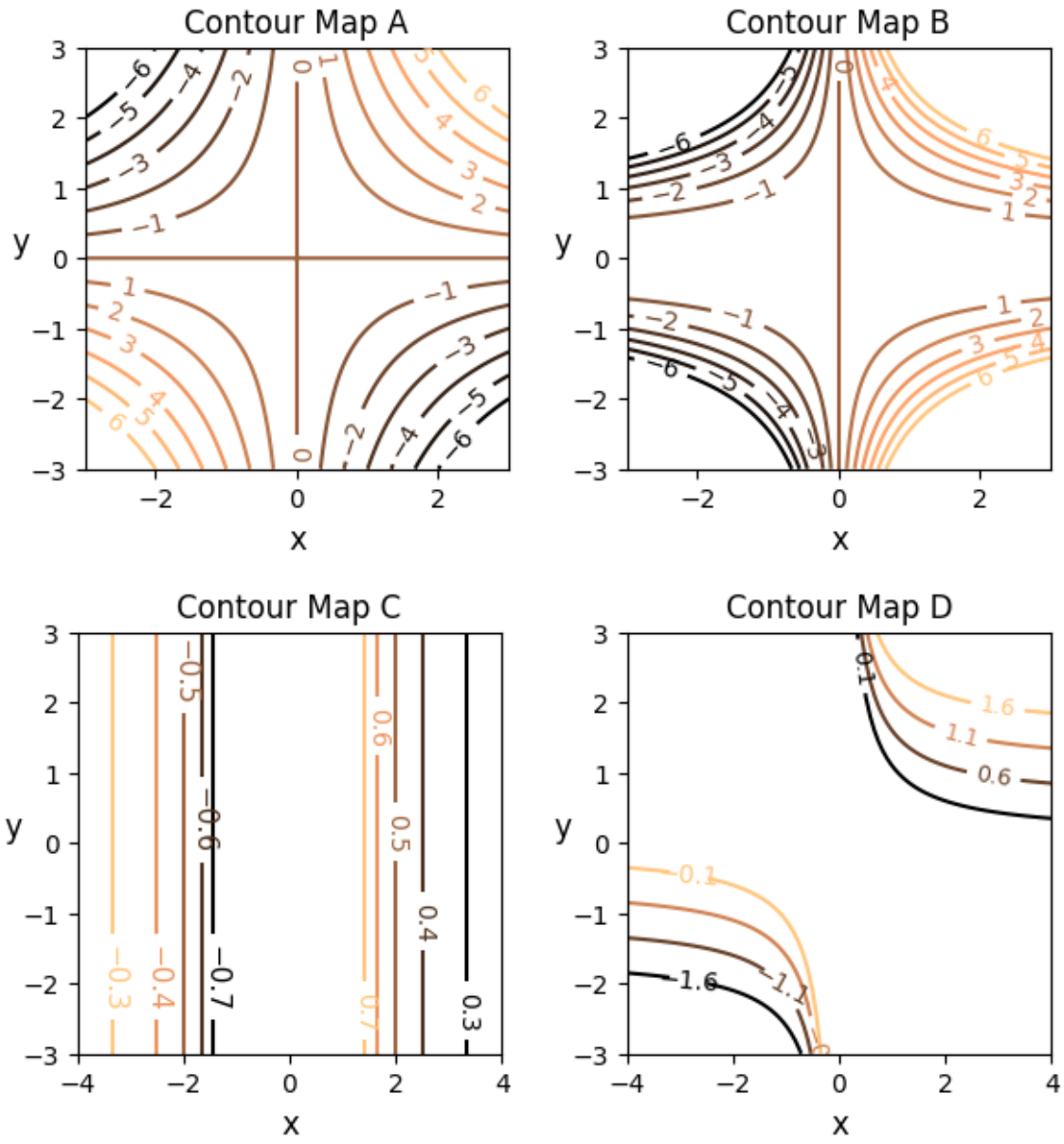

Figure 1: Matching contour maps

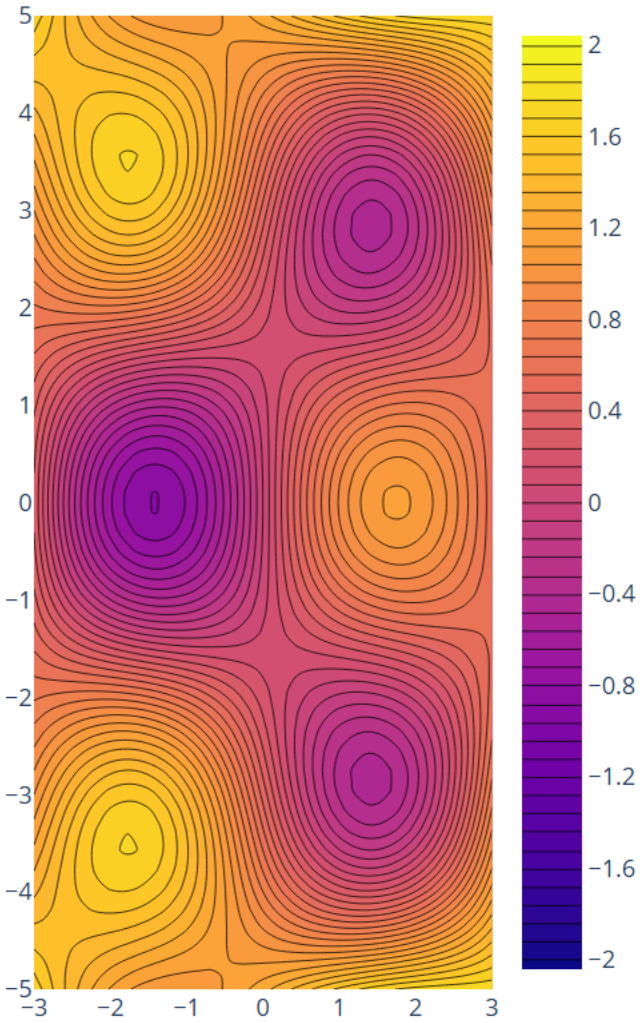

Figure 2: Contour map