Lecture 13

Goal for the next couple weeks: optimization of multivariable functions.
Examples of multivariable functions:

$f(x,y) = x + y$
$f(x,y) = \sin y$
$f(x,y) = 2$
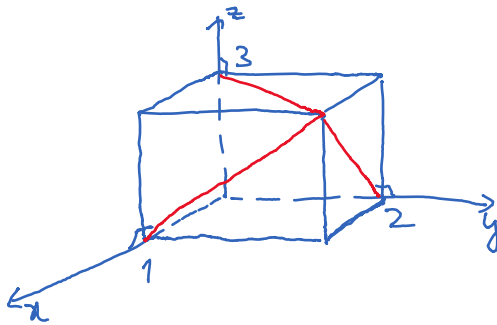$f(x,y,z) = yz + x$
$f(x,y,z) = xe^y$

Graph of a function $f(x)$ is the collection of points (x,y) on the plane such that $y = f(x)$. It is a curve.

Graph of a function $f(x,y)$ is the collection of points (x,y,z) in the space such that $z = f(x,y)$. It is a surface.

How do you locate the position of the point $(x,y,z) = (1,2,3)$?

You can graph a function on Python using the package **plotly** or **matplotlib**. The package matplotlib is more commonly used. However, it doesn't allow interaction with the graph such as zooming, dragging, rotating. To graph the function $f(x,y) = x^2 + y^2$ with **plotly**, use the following code:

```python
from plotly.graph_objects import*
from numpy import*
def f(x,y)
        return x**2+y**2
x = linspace(-5, 5, 50)
y = linspace(-5, 5, 50)
X, Y = meshgrid(x, y)
Z = f(X,Y)
fig1 = Figure(data=[Surface(z=Z, x=X, y=Y)])
fig1.show()
```

The *c-level set* (or *level curve*) of f is the collection of points (x, y) on the plane such that $f(x, y) = c$.

For example, consider the function $f(x, y) = x^2 + y^2$.
The 0-level set of f is the origin.
The 1-level set of f is the unit circle.
The 2-level set of f is the circle of radius $\sqrt{2}$.
The (-1)-level set of f is the empty set.
The collection of all level sets of f is called the *contour map*.

You can draw the contour map on Python with the **plotly** package as follows.

```
fig2 = Figure(data=Contour(z=Z,x=x,y=y))
fig2.update_layout(width=600,height=600)
fig2.show()
```