# Lecture 15

Wednesday, February 26, 2025 12:42 AM

Last time, you learned how to compute partial derivatives. Note that $f_x$ and $f_y$ are functions of $x$ and $y$ themselves.

$f_x(a,b)$ is the rate of change of the function $t \mapsto f(t,b)$ at $t = a$. Geometrically, $f_x(a,b)$ is the slope at $(a,b)$ of the curve obtained by intersecting the graph of $f$ with the plane $x = a$. In other words, it is the rate of change of $f$ at $(a,b)$ in the direction of the $x$-axis. Similarly, $f_y(a,b)$ is the rate of change of $f$ at $(a,b)$ in the direction of the $y$-axis.
*Gradient vector* is simply the vector consisting of all partial derivatives. It is the equivalence of derivative of a single-variable function.

**Example:**
$f(x,y) = x\sin(xy)$
$\nabla f(x,y) = (f_x, f_y) = (\sin(xy) + xy\cos(xy), x^2\cos(xy))$
**Example:**
$f(x,y,z) = xy + z^2$
$\nabla f(x,y,z) = (f_x, f_y, f_z) = (y, x, 2z)$

The gradient vector turns out to be all you need to compute the rate of change of $f$ in *any* direction. We need to make precise that "direction" means. Mathematically, it is a vector. A vector is an arrow with a length and a direction.

$u = (u_1, u_2)$ represents an arrow starting at the origin $(0,0)$ and ending at the point $(u_1, u_2)$. The length of $u$ is computed by the Pythagorean theorem:

$$|u| = \sqrt{u_1^2 + u_2^2}$$

We often need to compare two different directions. In that sense, we are interested in computing the angle between two vectors. It is surprisingly easy to compute (proof is a bit involved, using the Cosine Law of a triangle):

$$\cos\theta = \frac{u\cdot v}{|u||v|}$$

where $u \cdot v$ denotes the *dot product* of two vectors. It is computed as follows:

$$u \cdot v = u_1 v_1 + u_2 v_2$$

where $u = (u_1, u_2)$ and $v = (v_1, v_2)$.

Do a practice problem on the worksheet.