Lecture 17

Monday, March 3, 2025 2:30 AM

Goal for today: Gradient Descent/Ascent method for finding min/max of multivariable function $f(x, y)$.

Key observation: at a point (a, b),

- function f increases at the fastest in the direction of $\nabla f(a, b)$
- function f decreases at the fastest in the direction of $-\nabla f(a, b)$
- function is *level* in the direction orthogonal to $\nabla f(a, b)$

Gradient Ascent method for finding local maximum:
Start with an initial guess (x_0, y_0).
Then follow the direction of fastest rate of increase, which is $\nabla f(x_0, y_0)$:

$$(x_1, y_1) = (x_0, y_0) + \alpha \nabla f(x_0, y_0)$$

Here $\alpha > 0$ is called the learning rate. Then follow the direction of fastest rate of increase, which is $\nabla f(x_1, y_1)$:

$$(x_2, y_2) = (x_1, y_1) + \alpha \nabla f(x_1, y_1)$$

In general,

$$(x_{n+1}, y_{n+1}) = (x_n, y_n) + \alpha \nabla f(x_n, y_n)$$

This equation is in fact a couple of equations:

$$x_{n+1} = x_n + \alpha f_x(x_n, y_n)$$
$$y_{n+1} = y_n + \alpha f_y(x_n, y_n)$$

Gradient Descent method for finding local minimum: you follow the direction of fastest decrease instead, which is $-\nabla f$. The same recursive formula as above, except that the plus sign is replaced by the minus sign.

$$x_{n+1} = x_n - \alpha f_x(x_n, y_n)$$
$$y_{n+1} = y_n - \alpha f_y(x_n, y_n)$$

Practice on the worksheet.