Lecture 22

Monday, March 17, 2025 9:45 AM

Goal for the next two weeks: differential equations

A differential equation is an equation involving a function, its variable(s), and one or more of its derivatives.

Example:

$$y' + y = x$$
$$y'' = yy'$$
$$u_x = x^2 u_y$$

A natural phenomenon is usually modeled as a differential equation. A model is a simplification of a natural phenomenon in order for us to describe it in terms of equations.

Example:

1) Population growth: $y(t) =$ population at time t.
   You can imagine that a bigger population grows faster than a small population. It is reasonable to assume that the rate of change of population is proportional to the population itself. This argument produces a differential equation $y'(t) = \alpha y(t)$. Usually, it is written as $y' = \alpha y$. The solution is $y = ce^{\alpha t}$.
   This model is called Malthus' model (1798). It describes well the population for a short time. But it has a fatal implication for long time: that the population grows exponentially. Another model is proposed by Verhulst (1838) called *logistic model*: $y' = \alpha y(1 - y/M)$. Here, M is called the sustainable population. There are many more population models.
2) Heat transfer: $u_t = cu_{xx}$ where $u = u(x, t)$ is the temperature at position x at time t.
3) Wave propagation: $u_{tt} = cu_{xx}$