Lecture 28

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Artificial intelligence is a big field that includes machine learning, a computational method that mimic human learning process. Machine learning includes neural networks, a computational method that mimic the structure of the brain. Neural networks includes deep learning, which is a multi-layer neural network.

A neural network consists of inputs, one or more hidden layers, activated hidden layers, and outputs. The below figure is a simple neural network with input x, one hidden layer Z, activated hidden layer $\sigma(Z)$, and output y.

$$x \to Z \to \sigma(Z) \to y$$

Our brain perceives things through many layers of neurons. The very first layer ($x \to Z$) is when the input first gets into the brain. Z is a vector $Z = (z_1, z_2, \dots, z_H)$, where z_i is what the i'th neuron perceives about x. The dependence of Z on x is perhaps simple. So, we assume that Z depends linearly on x.

$$z_i = a_i x + b_i$$

Then the data z_i will be processed by neuron i. We model this process by applying a nonlinear function σ, called an *activation function*, to z_i. So, each z_i becomes $\sigma(z_i)$.

$$\sigma(Z) = \big(\sigma(z_1), \sigma(z_2), \dots, \sigma(z_H)\big)$$

Then the data processed by neurons $1,2, \dots, H$ will be synthesized together to an output, our single perception of x. The synchronization of neurons will be assumed to be linear. So,

$$y = c_0 + c_1\sigma(z_1) + c_2\sigma(z_2) + \cdots c_H\sigma(z_H)$$

The free coefficients $b_1, b_2, \dots, b_H, c_0$ are called *biases*. The coefficients of the "variables", namely $a_1, a_2, \dots, a_H, c_1, c_2, \dots, c_H$ are called *weights*. The total number of weights and biases is therefore $3H + 1$.