Lecture 5

Friday, January 24, 2025 9:36 PM

Standard form of a linear programming problem:

- must be a maximization problem,
- all linear constraints must be in a less-than-or-equal-to inequality,
- all variables are non-negative

Introduce slack variables:

- to turn all inequality constrains to equality constraints
- If the right hand side of any equation is negative, multiply that equation by -1.

Example:

Minimize $C = -2x + y$ subject to

$$x - y \le 3$$
$$2x + 3y \le 12$$
$$x + y \ge 2$$
$$x, y \ge 0$$

Convert the above linear programming problem into standard form and introduce slack variables.

Maximize $P = -C = 2x - y$ subject to

$$x - y + s_1 = 3$$
$$2x + 3y + s_2 = 12$$
$$x + y - s_3 = 2$$
$$x, y, s_1, s_2, s_3 \ge 0$$

Work on Problem 2 on the last worksheet.

Perform row operations:

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 3 \\ 2 & 3 & 0 & 1 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow[R_4=R_4+2R_3]{R_1=R_1-R_3,\ R_2=R_2-2R_3} \begin{bmatrix} 0 & -2 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 2 & 0 & 8 \\ 1 & 1 & 0 & 0 & -1 & 0 & 2 \\ 0 & 3 & 0 & 0 & -2 & 1 & 4 \end{bmatrix}$$

$$\xrightarrow[R_4=R_4+2R_1]{R_2=R_2-2R_1,\ R_3=R_3+R_1} \begin{bmatrix} 0 & -2 & 1 & 0 & 1 & 0 & 1 \\ 0 & 5 & -2 & 1 & 0 & 0 & 6 \\ 1 & -1 & 1 & 0 & 0 & 0 & 3 \\ 0 & -1 & 2 & 0 & 0 & 1 & 6 \end{bmatrix} \xrightarrow[R_3=R_3+R_2,\ R_4=R_4+R_2]{R_2=R_2/5,\ R_1=R_1+2R_2} \begin{bmatrix} 0 & 0 & 4/5 & 2/5 & 1 & 0 & 17/5 \\ 0 & 1 & -2/5 & 1/5 & 0 & 0 & 6/5 \\ 1 & 0 & 3/5 & 1/5 & 0 & 0 & 21/5 \\ 0 & 0 & 8/5 & 1/5 & 0 & 1 & 36/5 \end{bmatrix}$$

The last row reads $\frac{8}{5}s_1 + \frac{1}{5}s_2 + P = \frac{36}{5}$. Thus, $P = \frac{36}{5} - \frac{8}{5}s_1 - \frac{1}{5}s_2 \le \frac{36}{5}$. Then $C = -P \ge -36/5$.
The equality occurs when $s_1 = s_2 = 0$. Then $x = 21/5, y = 6/5$.

Example:

Maximize $P = 3x_1 + 2x_2 + 5x_3$ under the constraints

$$x_1 + 2x_2 + x_3 \ge 43$$
$$3x_1 + 2x_3 \le 46$$
$$x_1 + 4x_2 \le 42$$
$$x_1, x_2, x_3 \ge 0$$