Lecture 6

Monday, January 27, 2025 9:36 PM

Finish the example last time.
After **Step 1** and **Step 2**, we have turned the linear programming problem into:

Maximize $P = 2x - y$ subject to

$$x - y + s_1 = 3$$
$$2x + 3y + s_2 = 12$$
$$x + y - s_3 = 2$$
$$x, y, s_1, s_2, s_3 \geq 0$$

Step 3: write and manipulate the associated matrix to turn it into the following form:

$$\left[\begin{array}{cccccc|c} 1 & -1 & 1 & 0 & 0 & 0 & 3 \\ 2 & 3 & 0 & 1 & 0 & 0 & 12 \\ 1 & 1 & 0 & 0 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

want to keep these ≥ 0

want to make these ≥ 0

want to keep this column unchanged

Algorithm:

- Identify the key column. This is the column containing the "most" negative entry of the last row.
- Identify the pivot element on the key column. This is the element $a_j > 0$ such that the quotient b_j/a_j, where b_j is the far right coefficient on row j, is smallest. This guarantees that the coefficients on the right hand side is always nonnegative.
- Use elementary row operations to turn the key column into a column in which that pivot element becomes 1 and the rest become 0s.
- Go back to the first step. Stop when a desirable form is achieved: all coefficients on the left of the column of P on the bottom row are nonnegative.

After a number of row operations, we obtain the following matrix:

$$\begin{bmatrix} 0 & 0 & 4/5 & 2/5 & 1 & 0 & 17/5 \\ 0 & 1 & -2/5 & 1/5 & 0 & 0 & 6/5 \\ 1 & 0 & 3/5 & 1/5 & 0 & 0 & 21/5 \\ 0 & 0 & 8/5 & 1/5 & 0 & 1 & 36/5 \end{bmatrix}$$

This matrix corresponds to the following system of equations:

$$\frac{4}{5}s_1 + \frac{2}{5}s_2 + s_3 = \frac{17}{5}$$

$$y - \frac{2}{5}s_1 + \frac{1}{5}s_2 = \frac{6}{5}$$

$$x + \frac{3}{5}s_1 + \frac{1}{5}s_2 = \frac{21}{5}$$

$$\frac{8}{5}s_1 + \frac{1}{5}s_2 + P = \frac{36}{5}$$

Note that P (what we want to maximize) only appears in the last equation:

$$\mathrm{P} = \frac{36}{5} - \frac{8}{5}s_1 - \frac{1}{5}s_2 \le \frac{36}{5}$$

$\max P = 36/5$. The equality occurs when $s_1 = s_2 = 0$, which implies $x = 21/5, y = 6/5$.