Lecture 8

Friday, January 31, 2025 11:25 AM

The goal of the next three lectures is the *PageRank algorithm*, which is the original algorithm that Google used to rank websites for their search engine. To get there, you need to know some matrix algebra.

You can add or subtract matrices only if they have the same size. In that case, you add/subtract entry by entry. For example,

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} + \begin{bmatrix}0 & -2\\ 2 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 5 & 3\end{bmatrix}$$

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} - \begin{bmatrix}0 & -2\\ 2 & -1\end{bmatrix} = \begin{bmatrix}1 & 4\\ 1 & 5\end{bmatrix}$$

You can multiply any matrix by a number. For example,

$$A = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \qquad 2\mathrm{A} = \begin{bmatrix}2 & 4\\ 6 & 8\end{bmatrix}, \qquad -\mathrm{A} = \begin{bmatrix}-1 & -2\\ -3 & -4\end{bmatrix}$$

Multiplying a matrix by a matrix is a little more complicated. First, you need to know how to multiply a row by a column:

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}5\\ 6\\ 7\end{bmatrix} = 1(5) + 2(6) + 3(7) = 38$$

To be able to multiply a row by a column, they must be of the same length.
To multiply matrix A and matrix B, you want to be able to multiply each row of A by each column of B. This puts a restriction on the sizes of A and B: the number of columns of A must be equal to the number of rows of B.

Example:

$$A = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B = \begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

A is a 2×2 matrix, and B is a 3×2 matrix.
AB is not defined, but BA is well-defined.

$$\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}* & *\\ * & *\\ * & *\end{bmatrix}$$

In general, if A is an $m \times n$ matrix and B is an $n \times p$ matrix then AB is an $m \times p$ matrix.

Identity matrix: for each value of n, there is an identity matrix for each size n, denoted by I_n, which acts like number 1 in the regular multiplication.

$$I_1 = [1], I_2 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, I_3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Check that $AI_2 = I_2A = A$, $BI_2 = B$, I_2B undefined, $I_3B = B$.

Practice on the worksheet.

A *vector* is a matrix with only one row or only one column. For example,

$[1 \ \ 2]$ is a row vector of length 2

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a column vector of length 3

A nonzero column vector v is said to be a *stationary vector* of matrix A if $Av = v$.