

Worksheet 3/3/2025

Let $f(x, y) = x^3 + 3xy^2 - 3x$.

1) Find the gradient vector of f .

2) Write the recursion formula for the Gradient Descent method.

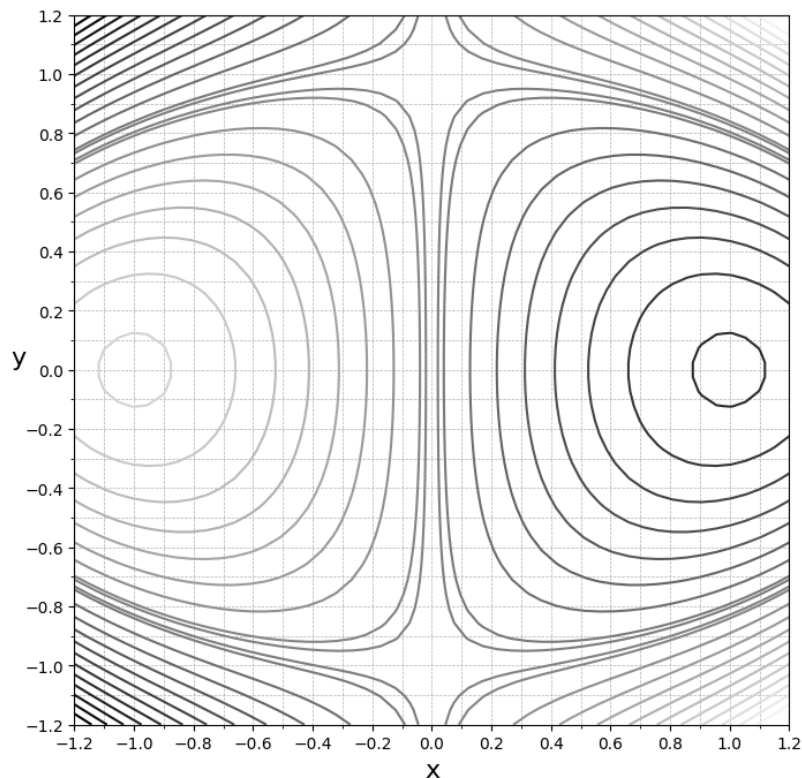
$$x_{n+1} =$$

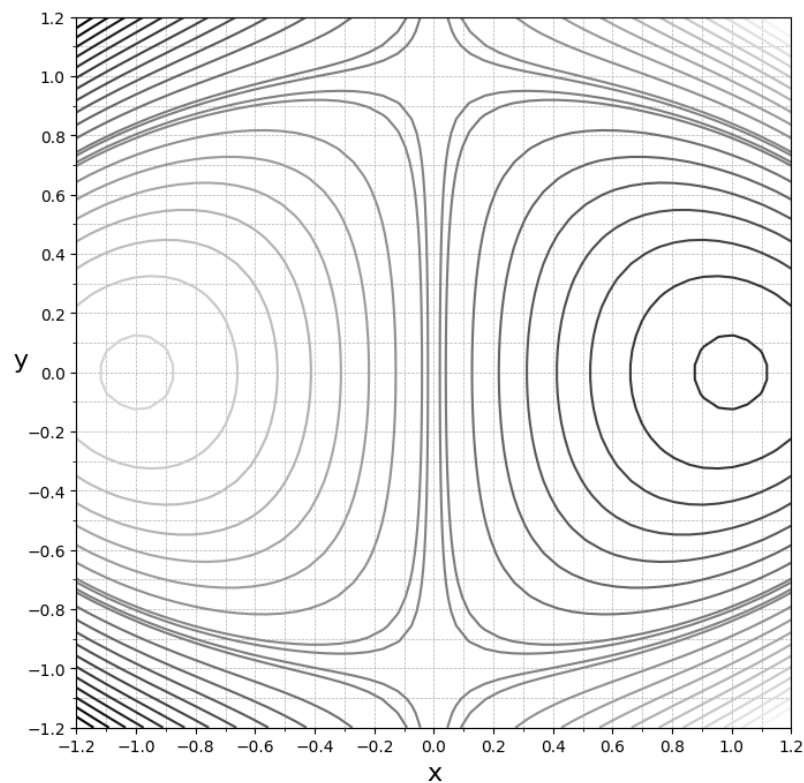
$$y_{n+1} =$$

3) With initial guess for local minimum $(x_0, y_0) = (-0.1, 0.5)$ and learning rate $\alpha = 0.15$, fill the following table using the Gradient Descent method. Round your results to 4 decimal places.

n	x_n	y_n	$f_x(x_n, y_n)$	$f_y(x_n, y_n)$
0				
1				
2				
3				
4				

4) Place the points (x_n, y_n) you found above in the contour map of f . Darker colors correspond to lower values.





Solve Problems 5-8 geometrically using the contour map.

- 5) Let $(x_0, y_0) = (-0.5, 0.5)$. If you use the Gradient Ascent method, what does the sequence (x_n, y_n) converge to?

- 6) Let $(x_0, y_0) = (0.5, 0.5)$. If you use the Gradient Ascent method, what does the sequence (x_n, y_n) converge to?

- 7) Let $(x_0, y_0) = (-0.5, 0.5)$. If you use the Gradient Descent method, what does the sequence (x_n, y_n) converge to?

- 8) Let $(x_0, y_0) = (-0.8, -1)$. If you use the Gradient Descent method, what does the sequence (x_n, y_n) converge to?