

Worksheet 6C

Normal distribution: a special symmetric, bell-shaped distribution, with a single peak. The normal distribution is completely identified by its mean and standard deviation.

68-95-99.7 rule: about 68%, 95%, 99.7% of all data fall within 1, 2, 3 standard deviations of the mean

Standard score (z-score) of a data value = the number of standard deviations above the mean

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

Percentile of a data value = the percentage of all data that are less than or equal to it.

1) State whether and why you would expect each of the following data sets to be normally distributed.

a) Scores awarded to a large class on an easy exam

b) The weights of bags of flour labeled “25 pounds”

c) The last digit of the Social Security number of 1000 randomly selected people

2) A set of test scores is normally distributed with a mean of 100 and a standard deviation of 20. Use the 68-95-99.7 rule to find the percentage of scores in each of the following categories.

(a) Greater than 100

(b) Less than 80

(c) Between 80 and 120

3) Find the z-scores and percentiles of the following data values in the normal distributions:

(a) 3.4 in the normal distribution with mean = 3 and standard deviation = 0.2

(b) 1.9 in the normal distribution with mean = 2.1 and standard deviation 0.3.

4) The SAT exams are designed so that their scores are normally distributed with a mean of 500 and a standard deviation of 100.

(a) Find the standard score and percentile of a score of 525. Explain the meaning of the percentile.

(b) Find the standard score and percentile of a score of 443. Explain the meaning of the percentile.

(c) Find the standard score and percentile of a score of 287. Explain the meaning of the percentile.

5) Scores on the verbal section of the GRE exam have a mean of 150 and a standard deviation of 8.5. Scores on the quantitative section of the exam have a mean of 152 and a standard deviation of 8.9. Assume the scores are normally distributed.

(a) Suppose a graduate school requires (among other qualifications) that applicants score at or above the 90th percentile on both exams. What verbal and quantitative scores are required?

(b) What percentage of students taking the verbal exam score above 160?

(c) What percentage of students taking the quantitative exam score above 160?

(d) Scores on the quantitative exam range between 130 and 170. What are the percentiles of these scores?

(e) Scores on the verbal exam range between 130 and 170. What are the percentiles of these scores?

(f) Students intending to study arts and humanities in graduate school have a mean score of 150 on the quantitative exam and a mean score of 157 on the verbal exam. Find the percentile of 150 on the quantitative exam. Find the percentile of 157 on the verbal exam.

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
-3.5	0.02	-1.0	15.87	0.0	50.00	1.1	86.43
-3.0	0.13	-0.95	17.11	0.05	51.99	1.2	88.49
-2.9	0.19	-0.90	18.41	0.10	53.98	1.3	90.32
-2.8	0.26	-0.85	19.77	0.15	55.96	1.4	91.92
-2.7	0.35	-0.80	21.19	0.20	57.93	1.5	93.32
-2.6	0.47	-0.75	22.66	0.25	59.87	1.6	94.52
-2.5	0.62	-0.70	24.20	0.30	61.79	1.7	95.54
-2.4	0.82	-0.65	25.78	0.35	63.68	1.8	96.41
-2.3	1.07	-0.60	27.43	0.40	65.54	1.9	97.13
-2.2	1.39	-0.55	29.12	0.45	67.36	2.0	97.72
-2.1	1.79	-0.50	30.85	0.50	69.15	2.1	98.21
-2.0	2.28	-0.45	32.64	0.55	70.88	2.2	98.61
-1.9	2.87	-0.40	34.46	0.60	72.57	2.3	98.93
-1.8	3.59	-0.35	36.32	0.65	74.22	2.4	99.18
-1.7	4.46	-0.30	38.21	0.70	75.80	2.5	99.38
-1.6	5.48	-0.25	40.13	0.75	77.34	2.6	99.53
-1.5	6.68	-0.20	42.07	0.80	78.81	2.7	99.65
-1.4	8.08	-0.15	44.04	0.85	80.23	2.8	99.74
-1.3	9.68	-0.10	46.02	0.90	81.59	2.9	99.81
-1.2	11.51	-0.05	48.01	0.95	82.89	3.0	99.87
-1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98